Activity 1B: Generating Sine and Cosine Graphs
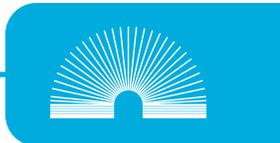
Students learn that sine and cosine waves can be used to represent tones. Students connect their prior knowledge of trigonometric ratios of acute angles within right triangles to the waveforms that result from graphing $y = \sin(x)$ and $y = \cos(x)$ on a coordinate system, using angle measure as the independent variable and the value of the trigonometric ratio in question as the dependent variable. Students create a cosine wave by hand, which allows them to focus on how the graph of the function is cyclical and appears as a wave and to better understand how the graph can model a sound wave.

Understandings

- Sound is a cyclical phenomenon, due to the nature of vibrations.
- Waves are a cyclical phenomenon.
- Since sine and cosine functions are also cyclical and have a wave graphical representation, these functions can be used to model sound.

Materials Needed

- Projection or printout of the graph in **Appendix A: Sound Wave Image**
- **Handout 3: Finding the Wave Function**
- Computers (one per pair of students, or one for the whole class with a projector) with Internet access
- Waveform applet (see *Advance Preparation*)
- **Handout 4: The Graph of Cosine**
- Graph paper
- Colored pencils
- Graphing calculators
- Rulers
- Protractors

1. Discuss the cyclical nature of sound.

Return to the observations students made during the first activity and emphasize the repetitious nature of the disturbances of air molecules. Tell students that the pattern of compression followed by rarefaction and back to compression of air molecules is *cyclical*, which means that it repeats at regular intervals as the sound source vibrates and the sound travels.

Show students a printout or projection of the visual representation of a simple sound wave in **Appendix A** and have them identify the following elements from the graph:

- The independent variable (time)
- The dependent variable (air pressure)
- Moments of high air pressure (crests)
- Moments of low air pressure (troughs)

Connect the crests and troughs with the "bubbles" that appeared in the Illuminations applet. The high-pressure moments are the compression moments, when the molecules are close together (the string presses them against each other), and this appeared in the applet as dense dots (where two bubbles met). The low-pressure (rarefaction) moments were when the molecules were farther apart, as the string moved away from the wave, and this appeared in the applet as an area with few dots (the center of each bubble).

Ask:

- What cycle do you notice in this graph?

  Possible answer: The graph shows a cycle of crest and trough, then another crest and trough.

- Do you notice any values that repeat in the graph?

  Possible answer: The values 1, 0, and –1 repeat, as do all the y-values in the graph.

Encourage students to share any other observations they have about the graph.

Remind students of two important ideas from the previous activity: *frequency* and *amplitude*. Ask them to suggest some ways that these two ideas appear in the graph. You don't need to verify or deny their suggestions, but collect as many ideas as you can. Tell them that these are two things they will figure out in this activity.

2. Have students explore the connection between circles and waves.

Give students **Handout 3: Finding the Wave Function** and have them work in pairs at computers.

Show students how to open the waveform applet by navigating to the Web site where it is located. Have pairs complete Handout 3.

As students work, especially on Step 3, listen to their conversations. Note which pairs or groups recall the trigonometric functions, even if they don't remember enough detail to identify the sine function as the one that uses the side opposite the angle.

3. Discuss the connection between the sine function and the wave.
Have students share some of their answers to the questions on Handout 3. In particular, be sure to discuss their ideas for the functions that can be used to represent the wave. Let them explain their thinking, even if their answers are incorrect. If necessary, provide hints to help them recall trigonometric functions.

Review how the sine and cosine functions work, and verify with students that the sine function would provide the y value of the wave form, at least when the wave is above the x-axis (and therefore the y value is positive).

Sketch a circle on the board with a right triangle inside it, as students saw in the applet. Label the triangle's sides (base, height, hypotenuse):
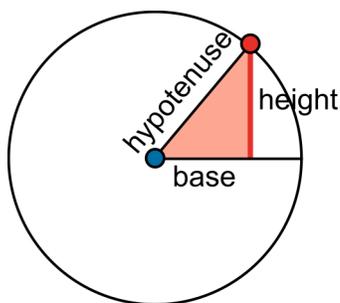
Show students that the sine and cosine of acute angles are ratios of the sides of the triangle. Explain that $y = \sin(x)$ and $y = \cos(x)$ represent functions whose independent variable is the measure of an angle and whose dependent variable is the ratio of the sides of the right triangle. Thus, as the measure of an angle changes, so do the values of these two ratios. Emphasize that corresponding acute angles of similar triangles will have the same function values.

Point out that the graph of the sound wave from **Appendix A** is also a graph of the basic sine function, $y = \sin(x)$.

4. Introduce radian measure and negative trigonometric values.

Point out that as the dot traveled around the circle in the applet, particular triangles were created. On a unit circle, when the distance traveled along an arc is equal to the length of the radius, the angle of rotation has a measure of 1 radian. When the angle of rotation is measured in radians rather than degrees, the arc length of a unit circle is equal to the measure of that angle. The angle of rotation can be used as the input value for sine and cosine functions instead of the measure of an acute angle in a right triangle.

Tell students that when they work with waves and other cyclical patterns, *radian measure*, rather than degree measure, is commonly used for trigonometric functions. Explain that it is always assumed that the rotation starts on the right (the 3 o'clock position) and rotates counterclockwise, as was shown by the dot moving around the circle in the applet.

> **Note:** If you wish, you can tell students how to convert degree measure to angle measure: Point out that as the dot travels the full circle, it sweeps out a 360° angle. Have students supply the distance traveled (the circumference of a circle with radius 1) and use that to consider half-circles (180° and π radians) and quarter-circles.

Point out that when using triangles to work with trigonometric functions, the values are always positive (because they are ratios of lengths), and the angles are always acute angles.

Have students use either the applet or the graph of the sound wave from **Appendix A** to identify what happens to the sine value when the angles become obtuse, and then when the dot travels to past the half-circle (and the *y* values become negative). Point out the connection between the sine of the angle of rotation and the *y*-coordinate of the point on a unit circle where the terminal ray intersects it.

Introduce the quadrants and how the sign of the trigonometric value depends on which quadrant the value of the independent variable (in this case, time) puts the dot.

Teacher's Notes: A Mnemonic for Helping Students Learn

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  II          I
  S           A
  ◄──────────►
  III         IV
  T           C
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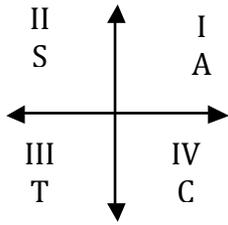
The following graphic and mnemonic device may be useful in helping your students remember the relationship between the quadrant in which a terminal ray lies and the sign of the angle's trigonometric values: ASTC (All Students Take Calculus).

In the first quadrant, all trigonometric functions have positive values.

In the second quadrant, sine and its reciprocal have positive values.

In the third quadrant, tangent and its reciprocal have positive values.

In the fourth quadrant, cosine and its reciprocal have positive values.

All trigonometric functions other than those named have negative values in each quadrant.

5. Have students generate the graph of cosine.
Give students **Handout 4: The Graph of Cosine**, along with graph paper, colored pencils, graphing calculators, rulers, and protractors. Have students complete Handout 4 individually or in groups. Tell students that this handout will give them a chance to work with cosine and see how it creates a wave graph.

> **Note:** Depending on students' recall of the trigonometric functions, you may first want to review special triangles (particularly 45°, –45°, and –90° triangles) and their cosine and sine values. You may also want to convert 45° to radians for or with the class.

Circulate to clarify the handout's instructions as students work.

6. Discuss Handout 4.
Assess students' understanding of Handout 4 by asking the following questions:

- What would happen to your graph of $y = \cos(x)$ if you continued cycling counterclockwise around the unit circle, using increasing angle measures?

  Possible answer: The graph would repeat, creating a continuous wave.

- What do you notice about the maximum and minimum values of the function?

  Possible answer: They range between –1 and 1.

- How long does it take for one cycle to be completed?

  Answer: One complete revolution around the circle, or 2π radians

7. Discuss the cyclical nature of the sine function.
Tell students that the change in angle measure (or time, when dealing with modeling sound waves) that it takes to complete one cycle is called the *period* of the function. Graphically, this translates into the horizontal width of one full wave between consecutive crests.

Ask:

- How are the graphs of sine and cosine function alike?

  Possible answer: They are both waves that reach a maximum value of 1 and a minimum value of –1. They have the same period—it takes 2 radians for both to complete one full cycle.

- How are the graphs of sine and cosine function different?

  Possible answer: They have a different starting value. At 0 radians, the value of sine is 0, while at 0 radians the value of cosine is 1, the maximum value.

- What is the period of the cosine wave you generated?

  Possible answer: The period is also 2π.

Tell students that both the sine and the cosine functions could be used to model sound. They will use the sine function in later activities.

> **Note:** In subsequent activities, students build on and transform the graphs of these parent functions to understand how changes in the wave equations affect the corresponding sound.