INTEGRATED MATHEMATICS UNIT

TEACHER GUIDE

LINEAR PROGRAMMING: OPTIMIZING MEDIA REACH

MATH

DIGITAL MEDIA ARTS

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Contents

Unit Overview

Unit Length

10 50-minute sessions

This unit brings together two disparate areas: creating the best possible publicity for a community event and the algebra of linear programming.

Students who study this unit may also be preparing to stage an art show (Foundations in Visual Arts and Media, Unit 7: Art Show!). They're developing promotional materials for the show and they want as many people as possible to come to their event. But what will bring in their audience—posters, newspaper ads, radio spots? How can they get the best bang for the buck?

Enter linear programming. Students may be surprised to find that mathematics can help them find answers to their publicity challenge.

This unit is built around a central problem: maximizing media reach to promote a youth media festival. Students are given a fictitious advertising budget and information about the cost and potential reach of newspaper and radio ads. Students determine how best to use these two different media to reach the greatest number of people for the least cost.

Students learn how to set up a linear programming problem and solve it graphically. To set up the problem, students create a mathematical model in which they

- *describe their goal in words and represent the goal mathematically as a linear expression called an* objective function
- *identify constraints, such as the cost of the ads and the timeframe for promotion, and represent these constraints as linear inequalities*
- *graph the linear expression and linear inequalities on the coordinate plane*

To solve the problem, students

- *find a feasible region in their graphs where all constraints are satisfied*
- *determine a solution within the feasible region that optimizes media reach, or finds the combination of newspaper and radio ads that reaches the greatest number of people for the least cost*

As students tackle this problem, the class also sets up and solves a simpler linear programming problem about maximizing profit. Students also work in pairs to formulate their own linear programming problem and discuss how they might solve it. Through their work on these optimization problems, students both experience the power of mathematical modeling and begin to understand its limitations.

Unit Portfolio

Students assemble a three-part portfolio of their work.

For Section 1, students record their work in solving a linear programming problem on choosing promotional media, Problem A—Media Selection.

For Section 2, students record their work in solving a linear programming problem about maximizing profit, Problem B—Profit Maximization.

For linear programming Problems A and B, students create the following:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- Problem resolution

For Section 3, students formulate and record ways to solve a linear programming problem they create with a partner. They also reflect individually on their work in the unit.

Assessment

Unit activities can serve as formative assessment tools. Use student work, including handouts, to gather information about student progress and to identify concepts or skills to reinforce within your instructional practice. The following are particularly useful for formative assessment:

- **Handout 8: Problem B—Profit Maximization: The Feasible Region** (Activity 2B.1)
- Students' formulation of their own linear programming problems (Activity 2B.3)

The problem-centered nature of the unit allows students to demonstrate their learning through authentic and relevant applications. For this unit, the summative assessment consists of:

- Three-section portfolio

The unit's Assessment Checklist provides criteria for assessment and a suggested weight for each. If you wish to use a rubric, work with same-grade-level or subject-area teachers to develop a tool that is consistent with your school's assessment system.

Framing Questions

- What do I need to know about a situation in order to formulate a useful problem?
- How can a mathematical model provide insights into a real-world problem?
- In particular, how can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost)?
- How can the linear programming process help me promote and set up an event such as a visual arts and media event?

Understandings

- We can use mathematical symbols to represent a real problem and then perform operations on the representation to gain more knowledge about the real problem.
- Linear programming can be applied to the promotion and set-up of a visual arts and media event to maximize audience reach, minimize set-up costs, or maximize profits from sales.
- Mathematical models are always oversimplifications but can provide useful insights into a situation.
- It is important to understand a problem well in order to work towards a solution. Sometimes a real problem is too complex to model mathematically, but with knowledge of the situation, sub-problems can be formulated and solved, providing logical implications for the original problem.

Where the Unit Fits In

This unit is designed as a stand-alone two-week student experience that can fit into a first- or second-year high school algebra course.

Integration with Foundations Courses

This unit integrates mathematical content and career and technical education (CTE) knowledge and skills. It can be taught before, at the same time as, or after the related unit in *Foundations in Visual Arts*.

Foundations in Visual Arts, Unit 7: Students create and stage an art show to present their artwork to a public audience. Students work in curation teams to create a theme and design for each section of the art exhibition. Students also work in exhibition preparation teams to prepare the exhibit space, design and create promotional materials, work on public relations tasks, and, optionally, manage a promotional budget. Students prepare artwork for the exhibition, ready the exhibition space, hang the artwork, and host an opening reception for a public audience. Discuss with the *Foundations in Visual Arts* course teacher the possibility of having students use the work they are doing to prepare for the art show as a basis for ideas for their own linear programming problems during Activities 1C.2 and 2B.3.

Multi-Disciplinary Teams

Use the following integrated units and integration suggestions for a school- or pathway-wide multi-disciplinary project.

***Going Public: Writing to Promote and Present Your Work* (English Language Arts).** Students develop promotional materials for an art show, including writing a press release and an artist statement.

Student Prerequisites

Prior to beginning the unit, students should:

- Be familiar with linear functions and their representations
- Be able to translate a written description of a problem into linear equations and inequalities
- Have some experience in solving systems of two linear equations in two unknowns

Pacing and Sequencing

You may need to build additional time into the unit in order to review concepts and skills with students. In Parts 1, 2, and 3, students work with linear equations, graph inequalities, and solve systems of linear equations.

You can find review material on these topics in the appendices:

- **Appendix A: More About Functions and Constant Rates of Change**
- **Appendix B: Working with Inequalities**
- **Appendix C: Solving Systems of Linear Equations**

Table of Activities

Part 1: Problem Design (4 sessions)

Students learn the meaning of linear programming and how it can be applied to solve real-world problems. They read about Problem A—Media Selection, a problem about choosing media to use in the promotion of a youth festival. Students work with partners to generate ideas for formulating their own linear programming problem.

Activity 1A: Media Selection

1A.1: Introduction to the Unit	Students are introduced to the problem-solving approach of linear programming and the unit activities.
1A.2: Student Reading: Problem A—Media Selection	In preparation for setting up a linear programming problem, students read about a youth media group and its efforts to promote public awareness for an upcoming event.
1A.3: What Is an Objective Function?	Students gain a conceptual understanding of an *objective function*, the expression that represents the optimization goal in a linear programming problem.

Activity 1B: Functions and Linearity

1B.1: Revisiting Problem A—Media Selection	Students read more about Problem A—Media Selection and use a graphic organizer to organize the information.
1B.2: Functions and Constant Rates of Change	Students write expressions and linear equations to represent information in Problem A—Media Selection. They graph linear equations on the coordinate plane and interpret the graphs' meaning in the context of the problem.

Activity 1C: Introduction to the Unit Portfolio

1C.1: The Unit Portfolio	Students are introduced to the required elements of their portfolios and receive a preview of assessment criteria.
1C.2: Partner Work	Students work with partners to brainstorm ideas for their own linear programming problem.

Part 2: Working with Constraints (3 sessions)

Students continue to learn about the problem-solving approach of linear programming by working with another, simpler optimization problem, Problem B—Profit Maximization. Students then apply the concepts learned in Problem B to Problem A—Media Selection.

Students represent both problems mathematically, graphing constraints as inequalities and finding the values of the decision variables that satisfy all of the constraints in each problem. This work prepares students for Part 3, where they find the optimal solution for both linear programming problems.

Activity 2A: Organizing the Constraints

2A.1: Making Sense of Constraints	Students are introduced to another linear programming problem, Problem B—Profit Maximization. They identify decision variables and constraints in the problem. Students represent the constraints mathematically as linear inequalities and work with partners to graph the inequalities on the coordinate plane.
2A.2: Constraints in Problem A—Media Selection	Students return to Problem A—Media Selection. The class creates a mathematical model for the problem by choosing decision variables, identifying an objective function, and representing the problem's constraints in terms of the decision variables.
2A.3: Partner Work—Problem Formulation	Students work in pairs as they practice formulating linear programming problems.

Activity 2B: The Feasible Region

2B.1: The Complete Graph and the Feasible Region: Problem B—Profit Maximization	Students work together to find all of the possible solutions, or the feasible region, for maximizing profit in Problem B—Profit Maximization.
2B.2: The Complete Graph and the Feasible Region: Problem A—Media Selection	Students work on their own to find the feasible region, or all the possible solutions for maximizing reach in Problem A—Media Selection.
2B.3: Partner Work	Student pairs formulate objectives and identify constraints in their own linear programming problem.

Part 3: Getting to a Solution (3 sessions)

Students determine how they can choose the best solution within the feasible region of a linear programming problem.

First, students use the objective function in Problem B—Profit Maximization to find the combination of drawings and collages that maximizes profit. Students then use the objective function in Problem A—Media Selection to find the mix of media vehicles that reaches the greatest number of people.

Finally, students examine whether it makes sense to implement these optimal solutions.

Activity 3A: How Can You Obtain the Best Solution?

3A.1: ***Optimal Solution: Problem B—Profit Maximization***	Students find the optimal solution to Problem B—Profit Maximization. Students see that the optimal solution to a linear programming problem occurs at one or more corner points of the feasible region.
3A.2: ***Optimal Solution: Problem A—Media Selection***	Students find the optimal solution to Problem A—Media Selection. They identify the corner points of the feasible region, find the coordinates of the point that maximizes reach, and determine whether the optimal solution makes sense in the real world.

Activity 3B: Completing the Unit Portfolio

Students assemble their portfolios and write a reflection about their work in the unit.

Advance Preparation

- Internet resources, provided as links in *Media & Resources*, are recommended throughout the unit for student or in-class use. These Web sites have been checked for availability and for advertising and other inappropriate content. Because Web site policies and content change frequently, however, we suggest that you preview the sites shortly before using them.
- Address any issues, such as firewalls, related to accessing Web sites or other Internet links at your school.
- Look at **Materials Needed** at the end of the unit and order any needed equipment or supplies.
- Many activities in the unit require students to manually create graphs of linear equations and linear inequalities. While the unit does not require the use of technological tools, The Geometer's Sketchpad® software and Texas Instruments graphing calculators can be used, if desired, to support the work in Part 2. See *Additional Resources for Teachers* for information about these tools.
- Look at the Appendices and decide whether you will include one or more of them as part of the unit. Go through the unit and plan your timing and lessons accordingly.
    - **Appendix A: More About Functions and Constant Rates of Change** gives a review of functions.
    - **Appendix B: Working with Inequalities** reviews one- and two-variable inequalities and their graphs on the number line and coordinate plane.
    - **Appendix C: Solving Systems of Linear Equations** can be used with students who have little prior experience in solving systems or who need support in making conceptual connections between the concepts involved and the process of elimination.
    - **Appendix D: Extension for Problem A—Media Selection** provides information for alternative formulations for the media selection problem.
- Encourage students to stay organized and to keep all of their work as they complete this unit. You may want to have students use their math notebooks (three-ring binders or spiral notebooks) to organize their portfolio work. Students can create three separate sections in their notebooks:
    - Problem A—Media Selection
    - Problem B—Profit Maximization
    - Problem formulation for my own linear programming problem and unit reflection

Part 1: Problem Design

Students learn the meaning of linear programming and how it can be applied to solve real-world problems. They read about Problem A—Media Selection, a problem about choosing media to use in the promotion of a youth festival. Students work with partners to generate ideas for formulating their own linear programming problem.

Length

4 50-minute sessions

Advance Preparation

- Before Activity 1B.2, create a completed graphic organizer that organizes the information from Problem A on Handout 4. See *Media & Resources* for links to blank graphic organizers, and see page 24 for a sample completed graphic organizer.
- Before Activity 1C.1, write the framing questions for the unit on chart paper:
  - What do I need to know about a situation in order to formulate a useful problem?
  - How can a mathematical model provide insights into a real-world problem?
  - In particular, how can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost)?
  - How can the linear programming process help me promote and set up an event such as a visual arts and media event?
- Before Activity 1C.2, gather examples of linear programming problems for students. (See *Media & Resources* for examples.)

RESOURCES

Activity 1A: Media Selection

Students read about ways to reach diverse audiences when advertising and promoting an event. This information sets up the first linear programming problem in the unit. Students examine the parts of the problem in preparation for the mathematics to follow.

Sequence

1A.1: Introduction to the Unit	Students are introduced to the problem-solving approach of linear programming and the unit activities.
1A.2: Student Reading: Problem A—Media Selection	In preparation for setting up a linear programming problem, students read about a youth media group and its efforts to promote public awareness for an upcoming event.
1A.3: What Is an Objective Function?	Students gain a conceptual understanding of an *objective function*, the expression that represents the optimization goal in a linear programming problem.

Understandings

- Linear programming is a mathematical problem-solving approach guided by the question, "How can I obtain the *best* solution?"
- An objective function serves to express the goal of obtaining the best possible solution given resource limitations.

Materials Needed

- **Handout 1: Unit Overview**
- **Handout 2: Problem A—Media Selection**
- **Handout 3: Objective Match-Up**

1A.1: Introduction to the Unit

1. Introduce the unit and go over the unit overview.
Distribute **Handout 1: Unit Overview** and have students read the introductory paragraphs to themselves. Review the sections *Unit Portfolio* and *What You Will Do in This Unit* with the class. Explain that students will explore how to apply the problem-solving approach of linear programming to real-world problems.

2. Draw attention to the vocabulary list.
Point out that Handout 1 contains many mathematical terms and media-related terms that students will use in the unit. Tell students that they can refer to this list when they encounter unfamiliar terms in unit activities.

Teacher's Notes: Vocabulary and Comprehension

This unit contains mathematical and media-related terminology that may be new to students. Take advantage of opportunities to use specific examples to clarify meaning for mathematical terms and to use context clues and other strategies to familiarize students with other key terms.

Handout 1:
Unit Overview

Optimizing Media Reach: Decision-making and Mathematics

As you prepare to stage a community event, either through your work in Foundations in Visual Arts and Media, Unit 7: Art Show! *or on your own, you want as many people as possible to attend. But what kinds of promotions will bring in your potential audience? Should you create posters or place ads in the newspaper or on the radio? You probably have only a limited amount of money to spend. How can you get the best bang for your buck?*

Enter mathematics—you may be surprised to learn that mathematics can help you find answers to your publicity challenge.

You'll work through a problem about maximizing media reach to promote a youth festival being organized by Wide Angle Youth Media, a non-profit organization. You'll be given an advertising budget and information about the cost and potential reach of newspaper and radio ads. You'll determine how to best use these two different media to reach the greatest number of people for the least cost.

You'll use a problem-solving strategy called linear programming and apply what you already know about linear equations and their graphs. By finding the best combination of newspaper and radio ads that reach the greatest number of people for the least cost, you'll find the answer to your publicity challenge.

Through your work on this and other problems, you will develop an understanding of the power of mathematical modeling and how you can use it to find solutions to real-world problems.

Your work in this unit will revolve around the following questions:

- *What do I need to know about a situation in order to formulate a useful problem?*
- *How can a mathematical model provide insights into a real-world problem?*
- *How can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost)?*
- *How can the linear programming process help me promote and set-up an event such as a visual arts and media event?*

Unit Portfolio

You will assemble a three-section portfolio of your work.

Section 1 will consist of your work with the class and a partner towards solving a linear programming problem about choosing promotional media. This is Problem A—Media Selection.

In Section 2, you will record your class and partner work in solving another linear programming problem. This is Problem B—Profit Maximization.

For linear programming Problems A and B, you will create:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- A resolution of the problem

In Section 3, you will record your work with a partner to formulate and explore ways to solve a linear programming problem that you create on your own. You will also use evidence from your work throughout the unit to respond to the unit's framing questions.

What You Will Do in This Unit

Find out what an objective function is. Explore objectives in real-world situations to prepare you for creating mathematical models to help solve real-world problems.

Set up a mathematical model for a linear programming problem. Organize and represent mathematically the information about Wide Angle Youth Media. Investigate how to determine the combination of newspaper ads and radio ads the group can use to achieve its objective: reaching the largest audience. Write the objective function in mathematical terms to maximize audience reach, and then graph it.

Learn about constraints in linear programming problems. Investigate a second linear programming problem related to Wide Angle Youth Media: what types of artwork should the group sell during its public event in order to maximize profit? Explore the limitations, such as cost of materials and quantity of artwork that the group can produce.

Find the best solution to each problem. Graph the constraints and use the graphs to determine how to achieve the optimum solutions of your objective functions. You'll find which combination of ads reaches the greatest audience and which combination of artworks yields the greatest profit.

Work with a partner to design your own linear programming problem. Come up with an idea for your own problem. Identify the objective function, the decision variables and constraints, and discuss ways to solve the problem.

Create a portfolio of your work. Document your work on the two linear programming problems, as well as your own problem. Reflect on your work in the unit.

Vocabulary Used in This Unit

Mathematics Terms

Constraints: In a linear programming problem, limitations of resources expressed as linear equations or linear inequalities.

Convex set: A set of points that contains an entire line segment joining any two of its points.

Decision variables: The unknown quantities that affect the objective function and the set of constraints, and frame the linear programming problem.

Dependent variable: A variable whose value *is determined by* the value of another variable.

Feasible region: The set of all ordered pairs that satisfy all of the constraints of a linear programming problem.

Function: A rule that maps each element in one set to exactly one element in a second set.

Independent variable: A variable whose value *determines* the value of another variable

Infeasibility: The situation in which no solution to a linear programming problem satisfies all of the constraints.

Linear combination: A sum of multiples of linear equations.

Linear equation: An algebraic equation in which each term is either a constant or a constant multiplied by the first power of a single variable. The standard form of a linear equation can be written: $ax + by = c$, where a, b, and c are constants, and x and y are variables.

Linear programming: A problem-solving approach that optimizes an objective function given limitations to resources (constraints). Both the objective function and the constraints can be represented with linear equations and/or linear inequalities.

Objective function: A mathematical expression that represents the goal of maximizing or minimizing a particular quantity when there are limited resources. The objective function is a measure of effectiveness that makes possible the comparison of feasible solutions.

Optimum solution: The feasible solution that provides the best possible value of the objective function.

Media-related Terms

Circulation: The total number of copies of a publication sold through various forms of distribution.

Exposure quality rating: A measure of the relative value of one advertisement in a specific medium. This measure takes into account factors such as audience demographics (e.g., age, income, education, race), image presented, and quality of the advertisement.

Media: The channels by which ads are carried to a target market.

Media objective: A statement in the media plan that explains the goals of the plan, often stating how many people within the target audience will be exposed to advertising messages in a given time period and how often.

Media plan/media schedule: A document that establishes how media will be used to disseminate the advertiser's message, including goals (objectives) and strategies.

Media reach: The percent of a target audience that can potentially be exposed to a particular media plan in a given time frame.

Media vehicle: Type of medium used to disseminate an advertiser's message. Some examples are radio, television, newspaper, posters, online sources, and bulletin boards.

1A.2: Student Reading: Problem A—Media Selection

In preparation for setting up a linear programming problem, students read about a youth media group and its efforts to promote public awareness for an upcoming event.

1. Have students read the problem.
Distribute **Handout 2: Problem A—Media Selection**.

Tell students that they will learn how to develop and apply a mathematical model to a real-world problem. Explain that the problem is about which types of media a group might choose to promote an event.

Tell students that they will revisit the information in Handout 2 throughout the unit as they learn about linear programming. Explain that linear programming is a problem-solving approach guided by the question, "How can you obtain the *best* solution?"

Give students time to read the problem on Handout 2.

> **Note:** You can read the problem together as a whole class or have students read it silently on their own.

2. Conduct a class discussion about the problem.
Ask students:

- What is the goal, or objective, of the Wide Angle Youth Media promotion committee?

  Answer: *To reach as many potential audience members for the festival as possible while staying within the given budget.*

Write the objective and display it for students, along with the list below:

- Newspapers
- Television
- Magazines
- Radio
- Cell phones
- Internet and the web, including email, web browsing, PC gaming
- Posters and bulletin boards
- Postal mail

Tell students that these are *media vehicles* that could be used to promote the festival in order to reach the objective. Explain that a media vehicle is a medium that is used to disseminate information or to communicate a message to a target audience.

Ask students how they might measure *reach* for each media vehicle. Discuss ways to quantify the number of people who could potentially see an ad in each medium.

Tell students that other factors also need to be taken into consideration when actually measuring media reach. For example, factors such as the cost of an ad in a newspaper or the range of a radio signal place limits on a newspaper's or radio station's reach. Explain that these factors can be thought of as *constraints*.

Tell students that in this unit they will learn a problem-solving approach known as *linear programming*. Explain that in a linear programming problem you use linear functions to represent an objective (such as reaching a potential audience) and constraints (such as working with a limited number of media vehicles and a budget).

Point to the objective and ask students:

- Can you think of a way to write the objective using mathematical terms? What are the unknown quantities? What are you *trying to determine*?

Handout 2:
Problem A—Media Selection

Introduction to Wide Angle Youth Media

Wide Angle Youth Media is a non-profit organization in Baltimore, Maryland. It provides media tools and education in video and audio production so that students can communicate messages to their community. Students produce and showcase their own work.

Wide Angle Youth Media reaches audiences in communities around the United States through online channels such as *Facebook* and *YouTube* and through partnerships with other cultural organizations. However, the audience in the local community is small, mostly composed of people directly connected to the organization, such as family members, board members, and community advocates.

Wide Angle Youth Media has taken on the challenge of widening its audience to reach more members of the Baltimore community. The group created a new mission and developed a festival to address the mission.

The *"Who Are You?"* Youth Media Festival is an event framed around the theme of *identity*. The festival is a collaboration among Wide Angle Youth Media and other nonprofit groups that serve youth in Baltimore. It includes not only video and audio, but also photography, fine art, poetry, live performances, and a gallery exhibit.

Promoting the Festival

Wide Angle Youth Media wants to reach beyond its usual audience and involve community members in the festival. So Wide Angle Youth Media has set up a promotion committee and a budget. The committee's task is to reach as many people as possible during the three weeks prior to the festival. Wide Angle's executive director wants to use only two types of media to promote the event.

As part of the promotion committee, you will help Wide Angle Media create a plan that *optimizes reach*—that is, reaches as many potential audience members as possible while staying within the given budget.

1A.3: What Is an Objective Function?

Students gain a conceptual understanding of an *objective function*, the expression that represents the optimization goal in a linear programming problem.

1. Define objective function as a class.

Tell students that the objective function in Problem A—Media Selection is an equation that represents the goal of maximizing reach. Display the more general definition for an objective function given below and have students record it in their notebooks:

> An **objective function** in a linear programming problem is a mathematical expression that represents the goal of maximizing or minimizing a particular quantity when there are limited resources.

2. Organize a Think-Pair-Share structure with a matching exercise.

Distribute **Handout 3: Objective Match-Up**. Tell students to read over Part 1. Have students "Think" and complete the matching exercise on their own. Then "Pair" students to talk about their responses.

> **Note:** Tell students that there may be more than one objective for a business. Students should be able to justify their choices.

Consolidate students' ideas in a whole class "Sharing" session.

3. Have students join together in a Think-Pair-Square.

Have pairs meet again and complete Step 2 on Handout 3. Students write an objective for each business (from Step 1 of the handout) and brainstorm possible limitations or constraints that might affect the objective of the business.

Teacher's Notes: Model Constraints

You may want to model one case so that students understand the kinds of limitations or constraints that are in effect when a business wants to optimize a certain quantity.

For example, present the case of an art gallery that wants to maximize profits from the sale of oil paintings and photographs. In this case, the objective is to maximize sales profit, while limitations might include the *cost* for the gallery's purchase of each piece of artwork, the *shipping cost* for transporting each piece, and the *labor* involved in framing the oil paintings and the photographs.

Have student pairs partner with another pair to form a team of four and share their responses with each other.

Then have student teams share their ideas with the whole class. Ask one student within each team of four to discuss the limitations for the objective of one of the given businesses.

4. Have students create a personal objective.
Ask students to generate a personal objective and record it on a sheet of paper. If needed, provide students with examples (see **examples** in *Teacher's Notes: Students' Personal Objectives*).

Ask volunteers to share their objectives with the class.

Teacher's Notes: Students' Personal Objectives

Students are likely to generate objectives that are not specific enough to allow them to create objective functions to represent them, or too difficult to write mathematically or to solve with linear programming. The goal here is to show that mathematical programming has numerous applications rather than to have students formulate easily solvable problems. However, you could use the following examples to show students how more specific information will allow them to create objective functions:

- *To maximize the profit of a basketball team that sells T-shirts and caps in order to raise money for travel:* $P = T(p1) + C(p2)$, where P is total profit, T is the number of T-shirts sold, C is the number of caps sold, $p1$ is the profit from selling a T-shirt, and $p2$ is the profit from selling a cap.
- *To minimize the cost of making cookies and brownies for a bake sale given that each baked good needs different amounts of available ingredients:* $C = K(c1) + B(c2)$, where C is the total cost, K is the number of cookies made, B is the number of brownies made, $c1$ is the cost of making a cookie, and $c2$ is the cost of making a brownie.
- *To maximize (or minimize) the time I spend playing video games given all the other tasks I need to accomplish in one day:* $T = 24 - (S + L + E + W + H + C)$ where T is time spent playing video games, S is time spent sleeping, L is time spent learning (in school), E is time spent eating, W is time spent washing up, H is time spent doing homework, and C is time spent doing chores (or working).
- *To determine the number of child and adult tickets to sell for the school play in order to maximize income:* $I = C(p1) + A(p2)$, where I is income, C is the number of child tickets sold, A is the number of adult tickets sold, $p1$ is the price of a child's ticket, and $p2$ is the price of an adult's ticket.

Handout 3:
Objective Match-Up

Part 1

Match each business or organization at the left with an objective at the right.

Business/Organization
United States Treasury
An airline
A car manufacturer
A coffee shop
A consumer
A mathematics department in a school
A hospital
A law firm

Objective
To maximize sales of a new model
To maximize knowledge and understanding
To maximize the number of surgical operations performed each day
To maximize tax revenue
To maximize the number of cases handled yearly
To maximize satisfaction through the consumption of goods and services
To minimize fuel costs
To maximize use of floor space

Part 2

For each organization in the left column of the table below, write its objective in the middle column. You can use the objective you identified in Part 1.

With a partner, brainstorm some limitations that might affect the objective of each organization. Write the limitations, or constraints, in the right column.

You might ask yourself, "What kinds of constraints or limitations could influence the objective of the business?" An example of constraints is provided below.

Organization	**Objective**	**Constraints**
An airline		
A car manufacturer		
A hospital		
Example: A consumer	To maximize satisfaction through the consumption of goods and services	A fixed income Prices of goods Quantity of goods available
A coffee shop		

Activity 1B: Functions and Linearity

Students are given additional information about Problem A—Media Selection. They organize the data and review linear functions in the context of the problem.

Sequence

1B.1: Revisiting Problem A—Media Selection	Students read more about Problem A—Media Selection and use a graphic organizer to organize the information.
1B.2: Functions and Constant Rates of Change	Students write expressions and linear equations to represent information in Problem A—Media Selection. They graph linear equations on the coordinate plane and interpret the graphs' meaning in the context of the problem.

Understandings

- Relationships between two variables that change with respect to one another can be mathematically represented with functions and their equations.
- The structure of a mathematical model is a useful tool for understanding and generalizing a problem-solving approach.

Materials Needed

RESOURCES

- **Handout 4: Problem A—Media Selection: More Information**
- Students' copies of **Handout 2: Problem A—Media Selection**
- Blank graphic organizer (one per group) (see *Media & Resources*)
- Completed graphic organizer containing information from Problem A on Handout 4 (see *Advance Preparation*)
- Graph paper (several sheets per student, pair, or group)
- Rulers (one per student, pair, or group)
- **Handout 5: Linear Functions and Their Representations**

1B.1: Revisiting Problem A—Media Selection

Students read more about Problem A—Media Selection and use a graphic organizer to organize the information.

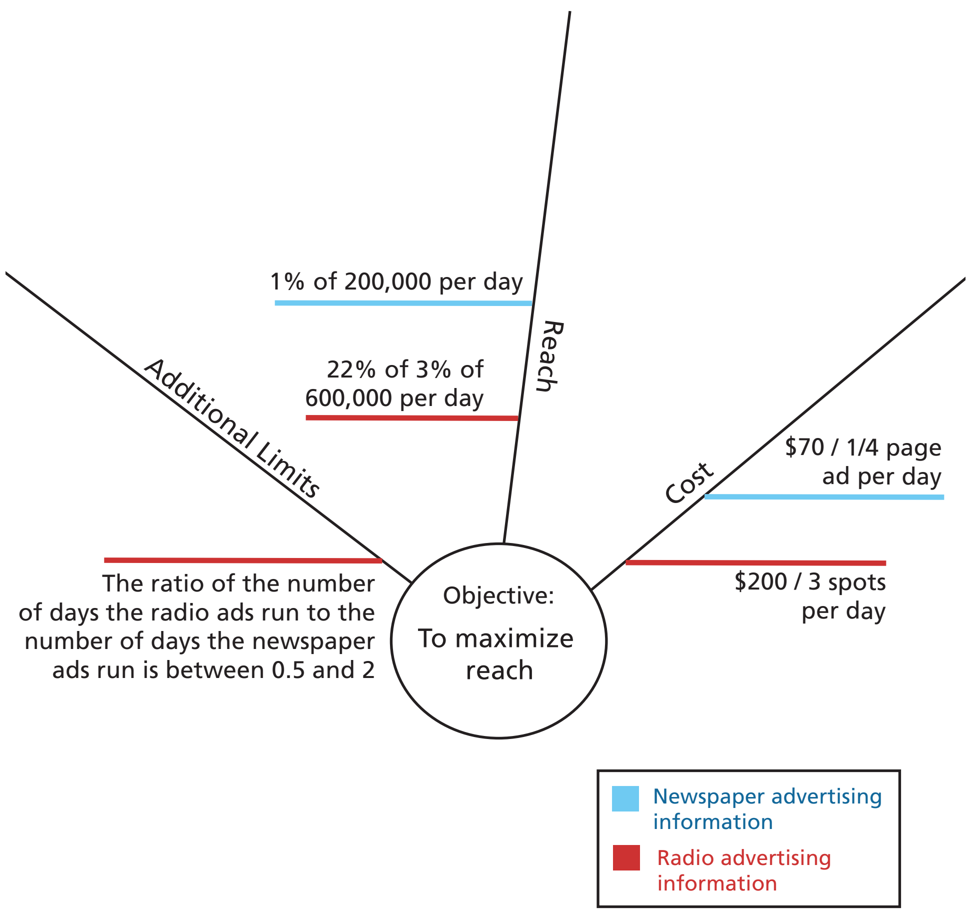
1. Distribute Handout 4: Problem A—Media Selection: More Information.
Have students read the additional information about the media selection problem provided in Handout 4. Students can refer to their copies of Handout 2: Problem A—Media Selection as they read.

2. Have students organize the information for Problem A.
Divide the class into groups of three and give each group a blank graphic organizer. Tell students the graphic organizer can help them structure all the information in the problem.

Have groups fill in their graphic organizers. Answer students' questions as needed.

Display a completed graphic organizer and discuss as a class.

Sample Completed Graphic Organizer



Handout 4:
Problem A—Media Selection: More Information

The Wide Angle Youth Media promotion committee for the "Who Are You?" Youth Media Festival wants to reach as many people as possible in the Baltimore area during the three-week period prior to the festival.

The Executive Director has allocated a budget for advertising. She has asked the promotion committee to limit advertising to two media vehicles, a local newspaper and a radio station. The promotion committee gathered the additional information below.

1. The advertising budget of $4,000 is to be used to promote the festival over the three-week period prior to the event.

2. For both newspaper ads and radio spot ads, the committee needs to know:
    - Number of potential customers reached through the medium
    - Cost per advertisement
    - Maximum number of times each medium is available during a time period
    - Audience demographics

3. Media kits on the Web sites for the newspaper and radio station give the cost of ads, as well as the number of potential customers reached through one ad.

    Local newspaper: A daily one-quarter page ad costs $70. The daily circulation is estimated at 200,000 people; the committee believes that only 1% of this number of people will respond to the ad and attend the festival for each day the ad runs.

    Local radio station: A set of three 30-second radio ads broadcast in one day costs $200. The total population of Baltimore is approximately 600,000. It is estimated that three daily broadcast spots reach about 3% of that population. Of this portion of the population, the committee expects that 22% will hear the radio ads and decide to attend the festival for each day the ads run.

4. Festival sponsors from the radio station and newspaper have placed one additional limitation on the use of the two media vehicles. They have asked that the number of days the ads run be distributed somewhat evenly between the two media. The limitation is described below:

    The ratio of the number of days the radio ads run to the number of days the newspaper ads run must be between ½ and 2. The sponsors recognize that if the ratio were 1, the limitation would be very restrictive; it would mean that the ads must run for the same number of days on each media vehicle. Instead, the sponsors are willing to be flexible by having this ratio range between ½ and 2.

1B.2: Functions and Constant Rates of Change

Students write expressions and linear equations to represent information in Problem A—Media Selection. They graph linear equations on the coordinate plane and interpret the graphs' meaning in the context of the problem.

Teacher's Notes: Linearity

Linear programming can only be applied to problems in which the objective function is expressed as a linear equation and the constraints as linear equations or inequalities.

Although these two requirements limit the use of linear programming, many relationships between variables in the real world can, in fact, be framed with linear equations. Throughout the unit, students apply linear programming to two-variable problems while they explore some of the limitations of the linear programming approach.

Mathematical methods and computer software have been developed to solve non-linear programming problems.

It is also possible to set up linear programming problems with more than two variables. Two variables implies working in two dimensions. It is beyond the scope of this unit to extend to three or more dimensions, although the methods used are simply conceptual extensions of the approach presented here (e.g., the simplex method).

1. Model how to write a function for Problem A—Media Selection.
Pose the following situation to students:

> *Suppose that the promotion committee wants to figure out the combinations of newspaper ads and radio spot ads to run over the three-week period prior to the event that will use the budget of exactly $4,000.*

> **Note:** Students can refer to the graphic organizers they created for Problem A—Media Selection and their copies of **Handout 2: Problem A—Media Selection** and **Handout 4: Problem A—Media Selection: More Information**.

Describe one approach to setting up this problem:

- ***Assign variables.***
  Work with students to determine the unknown quantities in the problem. Tell them to assign a letter to each unknown quantity, preferably a letter that reminds them of what the quantity represents.

Let r = number of radio advertisements (in sets of 3 per day).

Let n = number of newspaper ads.

- ***Look for relevant information.***
  Point out that this problem is only about the *budget*. Have students record the cost of a newspaper ad for one day and a set of three radio spot ads for one day.

  A set of three radio spot ads per day costs \$200.

  One quarter-page newspaper ad costs \$70.

- ***Write an equation.***
  Guide students to set up an equation. Ask:

  If one set of three radio spot ads costs \$200, how much do two sets of three radio spot ads cost? three sets? r sets? What expression can you write to represent the cost of placing r sets of three radio spot ads?

  Answer: $200r$

  If one newspaper ad costs \$70, how much do two newspaper ads cost? three newspaper ads? n newspaper ads? What expression can you write to represent the cost of placing n number of newspaper ads?

  Answer: $70n$

  Recall that the committee wants to find the number of advertisements in each medium that uses the given budget of \$4,000. Lead students to see that the equation that represents this goal (or objective) is:

  $200r + 70n = 4{,}000$

- ***Graph the equation using the intercepts.***
  Tell students to use the r- and n-intercepts to obtain a graph. They can change the form of the equation above (to slope-intercept form or point-slope form) in subsequent steps, as they interpret the meaning of their graph.

  For consistency, have the class choose an independent and a dependent variable. Throughout this unit, r is plotted on the x-axis and n plotted on the y-axis of a coordinate graph.

  When $r = 0$, the value of n can be found:

  $200(0) + 70n = 4{,}000$
  $70n = 4{,}000$
  $n = 57.14$
  Thus, the point (0, 57.14) is on the graph of this equation.

When $n = 0$, the value of r can be found:

$200r + 70(0) = 4{,}000$

$200r = 4{,}000$

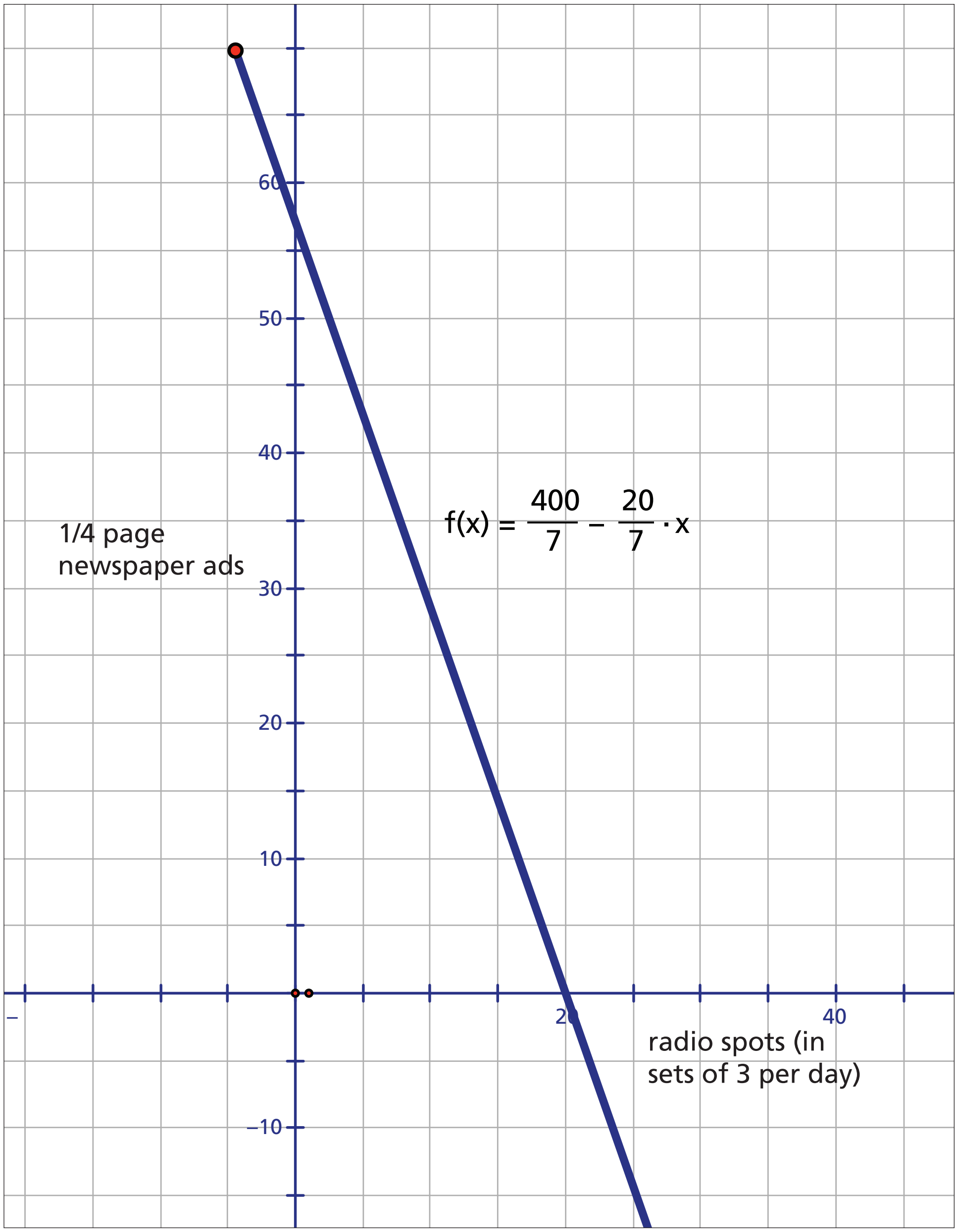
$r = 20$

Thus, the point (20,0) is on the graph of this equation.

Teacher's Notes: Continuous or Discrete?

Although the students have only two points to graph at this time, they are likely to connect them and form the line. In this case, the objective given is linear, so the graph represents the equation because two points determine a line. While the practical solutions to the problem posed must be integer solutions (making the equation a discrete function), it is still useful to temporarily ignore those limitations on the domain and range and think of it as a continuous linear function to simplify the graphing. You may want to discuss with students why this equation is linear, as well as the fact that all solutions of the equation may not be solutions of the problem posed. This issue will come up again near the conclusion of the unit, when the solutions they find are not integer solutions and other steps must be taken to find the practical solutions to the problem.

Below is a graph of 200r + 70n = 4,000.



2. Guide students through an interpretation of their work.

- Given the objective equation in this problem, can you say that '*n* is a function of *r*'? Why or why not?

  Possible answer: *Yes, as* r *changes (increases), n also changes (decreases).*

- Write an explicit equation of *n* in terms of *r*. In other words, solve for *n*.

  Answer:

  $200r + 70n = 4{,}000$
  $70n = 4{,}000 - 200r$

  $$n = \frac{400}{7} - \frac{20}{7}r = 57.14 - 2.86r \qquad (1)$$

- How many newspaper ads can be placed if the committee invests in 10 sets of three radio spot ads and uses the entire budget of $4,000? How many newspaper ads can be placed if the committee invests in 20 sets of three radio spot ads and wants to use the entire budget of $4,000? How many fewer newspaper ads can be placed *for each* additional set of three radio spot ads? What might this value mean in the context of the problem?

  Note: For the purposes of graphing this problem, you can assume that there are parts or fractions of an ad in both the newspaper and on the radio. For example, a fraction of a radio ad might be an ad that is shorter than a 30-second spot.

  Answer:

  If the committee invests in 10 sets of radio spot ads:
  $n = 57.14 - 2.86r$
  $n = 57.14 - 2.86(10) = 57.14 - 28.6 = 28.54$
  This means 28 one-quarter-page ads with an additional one-eighth-page ad.

  If the committee invests in 20 sets of 3 radio spot ads:
  $n = 57.14 - 2.86(20)$
  $n = 57.14 - 57.20 = -.06$
  A negative number of newspaper ads is not possible. So, 20 sets of three radio spot ads cannot be accommodated with the given budget.

  The number of newspaper ads that can be placed for each additional set of three radio spot ads can be found by calculating the number of newspaper ads needed when placing 11 sets of radio spot ads. This value can then be subtracted from 28.54, the number of newspaper ads placed for 10 sets of radio spot ads.
  $n = 57.14 - 2.86(11) = 57.14 - 31.46 = 25.68$

This means that for each additional set of three radio spot ads, you can place 2.86 fewer newspaper ads (28.54 – 25.68 = 2.86). Note that this is also the absolute value of the slope in equation (1).

- Is r a function of n? Why or why not?

  Possible answer: *Yes, as* n *changes (increases),* r *also changes (decreases).*

- Write an explicit equation of r in terms of n. In other words, solve for r.

  Answer:

  $200r + 70n = 4{,}000$

  $200r = 4{,}000 - 70n$

  $$r = 20 - \frac{7}{20}n = 20 - .35n \qquad (2)$$

- How many radio spot ads can be placed if the committee places 20 newspaper ads and wants to use the entire budget of $4,000? How many *fewer sets* of three radio spot ads can be placed *for each* additional newspaper ad placed?

  Answer:

  If the committee invests in 20 newspaper ads:

  $$r = 20 - \frac{7}{20}n$$

  $$r = 20 - \frac{7}{20}(20)$$

  $r = 20 - 7 = 13$ *sets of three radio spot ads*

  To answer the second question, students can either calculate the number of radio ads that can be placed when 21 newspaper ads are placed and compare this value to the number of ads that can be placed when 20 newspaper ads are placed, or they can think about the meaning of the slope of the linear equation above (2). As n *increases by 1, the equation takes away another* $\frac{7}{20} = .35$ *of a set of three radio spot ads.*

- What is the relationship between the number of newspaper ads that can be placed for an additional set of three radio spot ads and the number of sets of three radio spot ads that can be placed given an additional newspaper ad?

  Answer:

  At any time, if an additional set of three radio spot ads is placed, then $\frac{20}{7} = 2.86$ *newspaper ads must be given up. This is the slope of the linear equation with* n *expressed in terms of* r. *Because the slope is* *negative, you can place fewer newspaper ads for each additional set of three radio spot ads.*

At any time, if an additional newspaper ad is placed, then $\frac{7}{20}$ *sets of three radio spot ads must be given up. This is the slope of the linear equation with* r *expressed in terms of* n*. Because the slope is negative, you can place fewer radio spot ads for each additional newspaper ad.*

The two slopes, $-\frac{20}{7}$ *and* $-\frac{7}{20}$*, are reciprocals of each other.*

- If students haven't done so already, have them draw the line that passes through the *r*- and the *n*- intercepts. Ask them what they can say about all of the points that lie on the line.

  Answer:
  See graph on page 29. The points that lie on the line satisfy the equation and represent all of the combinations of radio spot ads and newspaper ads that use the entire $4,000 budget.

- Why does a graph of a *line* represent this equation?

  Answer:
  The graph is a line because the number of newspaper ads and the number of radio spot ads change with respect to one another at a constant rate.

- Could we have placed *n* on the *x*-axis and *r* on the *y*-axis? Why or why not?

  Answer:
  Yes. n *is a function of* r *and* r *is a function of* n *as well. Either variable can be considered the independent variable.*

- What is the solution to the problem?

  Answer:
  There are an infinite number of solutions to the equation, but not all are integer solutions! Any point on the line is a combination of newspaper ads and radio spot ads that costs exactly $4,000, but only three solutions are non-negative integer solutions, which is necessary for them to fit the constraints of the problem as posed. These solutions are:

  - *20 radio spot ads and zero newspaper ads*
  - *13 radio spot ads and 20 newspaper ads*
  - *6 radio spot ads and 40 newspaper ads*

Teacher's Notes: Constant Rates of Change

The approach presented here on linearity is intended to help students understand that the graph of an equation is a line when one quantity changes at a *constant rate* with respect to another quantity. If r increases by 1, n will always decrease by the same amount, regardless of the initial value of r.

Show students how a simple quadratic equation, such as $y = x^2$, does not have this property. Make sure they understand that, in this equation, the quantities x and y do not change at a constant rate with respect to each other.

Point out that in order to use the linear programming approach, both the objective function and the constraints must be able to be expressed as linear functions.

When students work on a graphical solution to linear programming problems later in the unit, they will be able to make a connection between linearity and the shape of the feasible region of solutions.

3. Distribute Handout 5: Linear Functions and Their Representations.
Have students continue to develop their concept of a function and their understanding of linearity. You may want to pair students or create small groups in which at least one student can provide guidance and mathematical support.

Provide students with graph paper and have them complete Handout 5. Tell them to refer to their copies of **Handout 4: Problem A—Media Selection: More Information** and their graphic organizers as they work on the problems in Handout 5.

Circulate to check students' work and understanding of concepts.

Note: Depending on students' previous knowledge of linear equations, you may want to do additional work with the equation of a line, particularly around the meaning of a line's slope. See **Appendix A: More About Functions and Constant Rates of Change**.

Once students have completed Handout 5, have them add any additional information they now have about Problem A (such as the actual number of people reached by an ad) to their graphic organizers.

Handout 5:
Linear Functions and Their Representations

You can represent the additional information in Problem A—Media Selection with expressions and linear equations. Use your knowledge of functions and linear equations to help you solve the problems below.

1. The promotion committee wants to understand further what the information in the newspaper media kit means. Consider the information given:

   A daily one-quarter page ad in the local newspaper costs $70. The daily circulation of the paper is estimated at 200,000 people; the committee believes that only 1% of this number of people will respond to the ad and attend the festival for each day the ad runs.

   Find the estimated reach (the number of people who may likely attend the festival if they see the newspaper ad) of a one-quarter page ad in the local newspaper.

   Possible answer:
   Under the best circumstances, 200,000 people will purchase the newspaper daily. 1% of this number will attend the festival. Thus, the estimated reach is 1% of 200,000.

   Estimated reach = .01(200,000) = 2,000 people.

2. Similarly, the promotion committee wants to understand further what the information in the radio station media kit means. Consider the information given:

   A set of three 30-second radio ads broadcast in one day costs $200. The total population of Baltimore is approximately 600,000. It is estimated that three daily broadcast spots reach about 3% of that population. Of this portion of the population, the committee expects that 22% will hear the radio ads and decide to attend the festival for each day the ads run.

   Find the estimated reach (the number of people who may likely attend the festival if they hear the radio spot ads) of a set of 3 radio spot ads broadcast in one day.

   Possible answer:
   Find 3% of the Baltimore population:
   .03(600,000) = 18,000.
   22% of this number will likely attend the festival. Thus the estimated reach is .22(18,000) = 3,960 people.

3. Use your findings from Problems 1 and 2 to determine the combination of newspaper ads and sets of radio spot ads that will reach exactly 20,000 people who will likely attend the festival.

- Assign variables. What are the unknown quantities? Use letters to represent these variables.

  Possible answer:
  Let r = number of sets of 3 radio spot ads
  Let n = number of newspaper ads

- Look for relevant information. What do you know about the estimated reach of each medium?

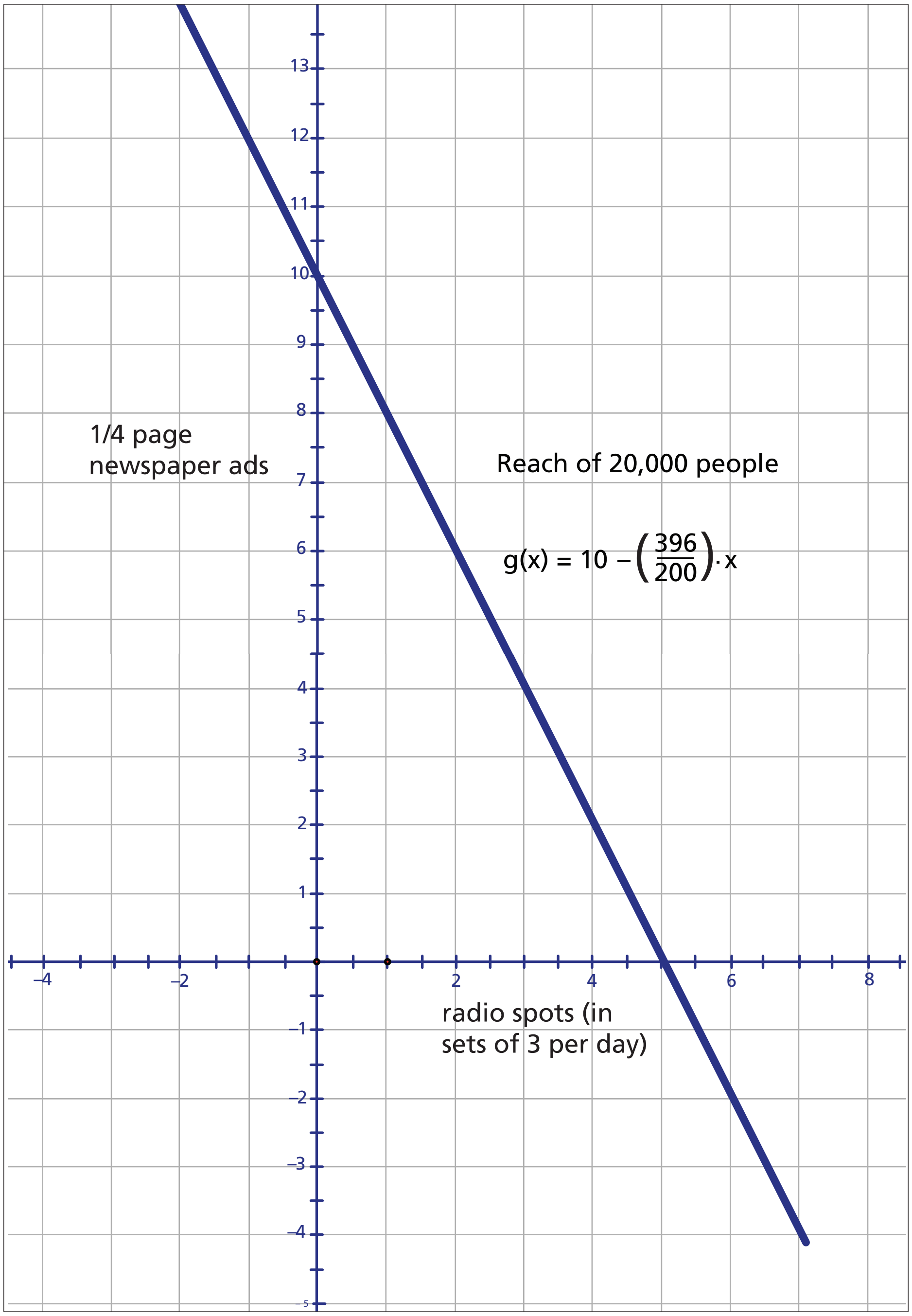
  Possible answer:
  One quarter-page newspaper ad will bring 2,000 people to the festival.
  One set of three radio spot ads run in one day will bring 3,960 people to the festival.

- Write an equation that represents the combinations of radio spot ads and newspaper ads that together reach exactly 20,000 people likely to attend the festival.

  Possible answer:
  r sets of radio spot ads bring $3{,}960r$ people.
  Similarly, n newspaper ads bring $2{,}000n$ people.
  To reach exactly 20,000 people, write the following equation:
  $3{,}960r + 2{,}000n = 20{,}000$

- Graph the equation using the intercepts.

  Answer:
  The graph of this linear equation is shown on the following page.



- Interpret the meaning of the graph.

  Possible answer: All points that lie on the line represent the combinations of newspaper and radio ads that will likely bring 20,000 people to the festival. Placing 10 newspaper ads and 0 sets of three radio spot ads has the same effect on reach as placing 0 newspaper ads and 5 sets of three radio spot ads.

Activity 1C: Introduction to the Unit Portfolio

Students learn about the requirements for their unit portfolios and meet with partners to come up with ideas for their own linear programming problems.

Sequence

1C.1: The Unit Portfolio	Students are introduced to the required elements of their portfolios and receive a preview of assessment criteria.
1C.2: Partner Work	Students work with partners to brainstorm ideas for their own linear programming problem.

Materials Needed

- **Handout 6: Assembling Your Portfolio**
- **Assessment Checklist: Unit Portfolio**
- Framing questions on chart paper (see *Advance Preparation*)
- Example linear programming problems (see *Media & Resources*)

1C.1: The Unit Portfolio

Students are introduced to the required elements of their portfolios and receive a preview of assessment criteria.

1. Review the requirements for the portfolio.
Give students **Handout 6: Assembling Your Portfolio** and review the unit portfolio requirements with the class.

2. Review the framing questions.
Display the chart paper with the unit framing questions that you prepared. Tell students that they will respond to these questions in their portfolios, using concepts, ideas, and examples from their work throughout the unit.

> **Note:** You may want to post the framing questions in the classroom so that students can easily refer to them as they work.

3. Distribute the assessment checklist.
Go over the Assessment Checklist: Unit Portfolio with students. Answer any questions students may have.

Handout 6:
Assembling Your Portfolio

You will learn the principles behind linear programming as the class works on Problem A—Media Selection. You will also apply the linear programming approach to another problem, Problem B—Profit Maximization. In addition, you will work with a partner to formulate your own linear programming problem and discuss ways to solve it.

You will assemble all of your work in a three-section portfolio.

Portfolio Requirements

Section 1: Problem A—Media Selection

Gather work done on Problem A—Media Selection. Be sure to include:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- Resolution of the problem

Section 2: Problem B—Profit Maximization

Gather work done on Problem B—Profit Maximization. Be sure to include:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- Resolution of the problem

Section 3: Problem Formulation and Written Reflection

Document your work with a partner on an idea for your own linear programming problem. Be sure to include:

- A problem statement
- A statement of the objective function
- Your choice for possible decision variables
- Constraints in terms of the decision variables chosen

Write a response to the framing questions below, using examples from the work you did throughout the unit:

- What do I need to know about a situation in order to formulate a useful problem?
- How can a mathematical model provide insights into a real-world problem?
- In particular, how can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost) that is dependent on variables I can change?

Assessment Checklist:
Unit Portfolio

Use this checklist to help you assemble your portfolio. Make sure to include all the requirements. Your teacher will use this assessment to evaluate your work.

Requirements	Percentage of Total Grade	Comments	
Section 1. Problem A—Media Selection	**40%**	**Student Comments**	**Teacher Comments**
Mathematical model includes an explanation of the decision variables.			
The objective function is a linear expression stated in terms of the decision variables.			
The set of constraints is expressed as linear inequalities in terms of the decision variables.			
A graphical solution displays the feasible region.			
Corner points are identified and the *best* solution determined.			
Section 2. Problem B—Profit Maximization	**40%**	**Student Comments**	**Teacher Comments**
Mathematical model includes an explanation of the decision variables.			
The objective function is a linear expression stated in terms of the decision variables.			
The set of constraints is expressed as linear inequalities in terms of the decision variables.			

A graphical solution displays the feasible region.			
Corner points are identified and the *best* solution determined.			
Section 3. Problem Formulation and Written Reflection	**20%**	**Student Comments**	**Teacher Comments**
Partner work: The problem statement generated can be resolved by using a linear programming approach.			
Partner work: The objective is clearly explained and justified.			
Partner work: The objective and the constraints depend on two decision variables.			
Individual reflection: Evidence from work is used to support responses to the framing questions.			
Individual reflection: Response is well organized and addresses the framing questions clearly and thoroughly.			
Total	**100%**		

1C.2: Partner Work

Students work with partners to brainstorm ideas for their own linear programming problem.

1. Introduce the activity.

Tell students that they will work in pairs to create a linear programming problem of their own. Assign each student a partner.

Teacher's Notes: Pairing Students

Student partners should be:

- compatible in their concept understanding and skill development in mathematics
- willing to provide support to one another
- ready to learn together and hold each other accountable

If students are also preparing to stage the art show in the course *Foundations in Visual Arts, Unit 7: Art Show!*, you may want to pair students who are members of the same exhibition preparation team so that they can use their work in the course as a basis for ideas for their own linear programming problem.

2. Give students time to brainstorm ideas for a problem of their own.

Give students 15 minutes to brainstorm ideas for their own linear programming problems. Explain that they can create fictional scenarios or they can adapt an already formulated linear programming problem. Tell students to record their ideas in their notebooks.

Note: See *Media & Resources* for examples of linear programming problems you can provide to students. Students also work with example linear programming problems in Activity 2A.3 when they practice formulating problems.

Since students' understanding of constraints is limited at this point, tell students that their problems should focus on an *objective function* that depends on two variables.

Part 2: Working with Constraints

Students continue to learn about the problem-solving approach of linear programming by working with another, simpler optimization problem, Problem B—Profit Maximization. Students then apply concepts learned in Problem B to Problem A—Media Selection.

Students represent both problems mathematically, graphing constraints as inequalities and finding the values of the decision variables that satisfy all of the constraints in each problem. This work prepares students for Part 3, where they find the optimal solution for both linear programming problems.

Length

3 50-minute sessions

Advance Preparation

RESOURCES

- Before Activity 2A.1, decide whether you will have student pairs use sheets of graph paper or gridded transparencies to graph the constraints in the linear programming problem. You may want to have students graph each constraint on a separate transparency in preparation for identifying the feasible region in Activity 2B.
- Make enough transparencies so that you can distribute at least four to each pair. You can create the transparencies by printing a graph-paper grid template onto printer-friendly transparencies. Have transparency markers available for students to write on the transparencies. If students are instead using sheets of graph paper to create their graphs, have colored pencils available.
- Before Activity 2A.3, choose several different linear programming problems to use as examples. Make enough copies so that you can distribute one problem to each pair of students. (It's fine if some pairs work with the same problem, as long as each pair has its own copy to work with.) (See *Media & Resources* for example problems.)

Activity 2A: Organizing the Constraints

Students work with a different linear programming problem, Problem B—Profit Maximization. They represent the problem mathematically and graph the constraints as linear inequalities. They apply what they've learned about decision variables and constraints to Problem A—Media Selection. Student pairs work together to represent another linear programming problem mathematically.

Sequence

2A.1: Making Sense of Constraints	Students are introduced to another linear programming problem, Problem B—Profit Maximization. They identify decision variables and constraints in the problem. Students represent the constraints mathematically as linear inequalities and work with partners to graph the inequalities on the coordinate plane.
2A.2: Constraints in Problem A—Media Selection	Students return to Problem A—Media Selection. The class creates a mathematical model for the problem by choosing decision variables, identifying an objective function, and representing the problem's constraints in terms of the decision variables.
2A.3: Partner Work—Problem Formulation	Students work in pairs as they practice formulating linear programming problems.

Understandings

- Constraints in a linear programming problem have a direct effect on the decision variables that define the objective function.
- Linear inequalities can be used to represent constraints in a linear programming problem.

Materials Needed

- **Handout 7: Problem B—Profit Maximization**
- Graph paper or transparencies with grids (see *Advance Preparation*)
- Rulers
- Colored pencils or transparency markers (see *Advance Preparation*)
- Students' copies of **Handout 2: Problem A—Media Selection**

- Students' copies of **Handout 4: Problem A—Media Selection: More Information**
- Students' work on Handouts 2 and 4 (from Activities 1A.2 and 1B.1)
- Students' completed copies of **Handout 5: Linear Functions and Their Representations**
- Example linear programming problems (one problem per pair—see *Advance Preparation*)

2A.1: Making Sense of Constraints

1. Introduce the concept of decision variables.
Distribute **Handout 7: Problem B—Profit Maximization**. Tell students that they are now going to work on another linear programming problem, Problem B—Profit Maximization. Explain that they will work in pairs to create a mathematical model for the problem.

Have students read the introductory paragraph on Handout 7. Explain that they will learn about the decision variables and constraints in a linear programming problem. Then have students read the section on the handout called *The Problem* so that they have an example in mind as you introduce the concept of decision variables.

Explain that businesses use linear programming to maximize or minimize a particular quantity. For example, a business may want to determine how much of two items to manufacture in order to maximize profit or minimize production costs.

Tell students that the first step towards solving a linear programming problem is to identify the *decision variables*. Stress that this must be done even before the objective function of the problem can be written. Decision variables are the unknown quantities that affect both the objective function and the set of constraints in a linear programming problem.

Use the scenario of a business whose objective is to maximize profit from the production of two items. Point out that the number of each item that the business can produce is a decision variable. Tell students that you can label the two decision variables x and y. Then you can write the objective function *in terms of x and y*. Recall with students that the objective is to maximize profits.

Explain that when you choose the decision variables, you restrict the linear programming problem by narrowing it and simplifying it. Yet choosing the variables also gives you a way to resolve the problem.

Point out that the constraints in the problem also need to be written in terms of the decision variables. In the scenario of the business trying to maximize profits, the constraints are the limited resources that are needed to produce each of the two items.

Copy the table below onto the board. Explain to students how the table can be used to organize the constraints in a linear programming problem.

Tell students that Products X and Y are represented by the decision variables *x* and *y* and that Inputs A, B, and C represent the resources (such as materials, labor, time) needed to produce each product. The column on the right, Quantities of Input Available, is used to show that resources are limited.

Post the Table of Constraints in the classroom so that students can refer to it for their work throughout the unit.

Table of Constraints			
	Product X ***x***	**Product Y** ***y***	**Quantities of Input Available**
Input A	Quantity of input needed to produce product	Quantity of input needed to produce product	Availability of resource
Input B	Quantity of input needed to produce product	Quantity of input needed to produce product	Availability of resource
Input C	Quantity of input needed to produce product	Quantity of input needed to produce product	Availability of resource

2. Have pairs begin work on Handout 7.

Teacher's Notes: Scaffolded Instruction

Handout 7 presents students with Problem B—Profit Maximization, a less complex linear programming problem than Problem A—Media Selection. Using a simpler, yet similar, problem is one way the material in this unit is scaffolded for students.

Problem A is complex and contains a lot of information. By working with Problem B, students can better understand the parts of the problem-solving approach. Problem B also uses smaller numbers, which enables students to create graphs more easily.

As a class, review the introductory paragraph and the problem on Handout 7. Pair students with their partners to begin work on the handout. Explain that they will work together to create a mathematical model for the problem. Tell them that they will use this model later in the unit to find the best solution to the problem of maximizing profit.

Pairs can use one handout to show their work. Have pairs complete Step 1 together.

Check students' work after they have set up the problem in Step 1. Make sure that they have correctly identified the objective function, have assigned variables to the unknown quantities, and have noted each constraint in the problem.

Teacher's Notes: Assigning Variables

Students might assign the decision variables differently from the way the variables are assigned in the answers for Step 1 on Handout 7. This is okay. Just note that the solutions shown are not the only correct answers.

Check to make sure that students are explicit about which variable is the independent variable and which is the dependent variable, as this makes a difference in the setup of the graphical solution. It does not matter how students assign the variables, as long as students keep their use consistent throughout the problem.

Tell students that now that they have set up the linear programming problem, they are going to create a Table of Constraints for the problem. Have students complete Step 2 on Handout 7. Check that pairs have organized the constraints correctly.

3. Lead a whole-class discussion on solving inequalities.

> **Note:** Before working with constraints and linear inequalities, you may wish to review one- and two-variable inequalities and how to graph them on the number line and the coordinate plane. You can use **Appendix B: Working with Inequalities** for a review.

Prepare students for working with each constraint in Step 3 on Handout 7 by looking at linear inequalities. Use Constraint 1, Time, and work through the process on Handout 7 together with students.

Teacher's Notes: Technology Tools

You may want to have students use technology tools, such as The Geometer's Sketchpad® software or Texas Instruments graphing calculators, as they work with linear inequalities. See *Additional Resources for Teachers* for information about these tools.

4. Have students complete Handout 7.

Tell students that now that they have set up the linear programming problem and created a Table of Constraints, they are going to work with each constraint in the problem.

Explain to students that once they have completed this handout, they will have formulated a linear programming problem and graphed each constraint. Tell them that later they will use their work on Handout 7 to find the best solution to Problem B, that is, which combination of drawings and mixed-media collages yields the greatest profit.

Distribute graph paper or gridded transparencies to student pairs. Have pairs complete Step 3 on Handout 7.

Teacher's Notes: Logistics for Handout 7

Students' work on Handout 7 prepares them for the next step in resolving Problem B—Profit Maximization. The next step is to identify the feasible region, or all the possible solutions to the problem.

Students graph each of the four constraints on the coordinate plane. Below are some options you can use for student graphing.

- Students graph each constraint on a separate gridded transparency and use markers to shade each constraint. Students can then overlay the transparencies in the next step when they generate the feasible region.

- Students graph each constraint on a separate sheet of graph paper and use different-colored pencils to shade each constraint. Students can then consolidate the graphs onto one coordinate plane when they generate the feasible region.

Help students scale the axes on their coordinate planes so that the axes accommodate the range of each decision variable for the problem. Be sure to tell students to use the same scale for each graph they create. This will allow them to easily consolidate the graphs when they generate the feasible region.

Have pairs check their work with another pair of students.

Have students set aside their work on Handout 7. Tell them they will return to it later in the unit when they continue their work on resolving Problem B—Profit Maximization.

Explain that next they will turn to Problem A—Media Selection and work with constraints in that linear programming problem.

Handout 7:
Problem B—Profit Maximization

You and your partner are going to create a mathematical model for a linear programming problem about maximizing profit. You are going to:

- determine the decision variables
- write the objective function in terms of the decision variables
- identify the constraints on the decision variables
- organize the constraints in a table
- graph the inequalities that represent each constraint

Read the linear programming problem below and then follow the steps to work towards a solution for the problem.

The Problem

You plan to sell two kinds of artwork at the Youth Media Festival and donate the proceeds to Wide Angle Youth Media for future work in youth media education.

You want to figure out how many drawings and how many mixed-media collages to make prior to the festival. You can spend up to $240 on supplies. The cost of supplies for one drawing is $8. The cost of supplies for one mixed-media collage is $16. You have enough time to complete at most 20 pieces of artwork.

If each drawing makes a profit of $40 and each mixed-media collage makes a profit of $65, how many of each type of artwork should you create to maximize the amount of money received from sale of the artwork?

Step 1: Set up the problem.

1. State the objective of the problem in your own words.

   Possible answer: The objective is to determine the number of mixed-media collages and the number of drawings to make in order to maximize the amount of money received from sales.

2. The unknown quantities in this problem are the number of drawings and the number of mixed media collages. Assign the variable *x* to represent the number of drawings and *y* to represent the number of mixed media collages.

3. How are your decision variables related to each other?

   Possible answer: One is a function of the other. The number of drawings I make determines the number of mixed-media collages I make and vice-versa.

4. Write your objective in terms of your decision variables. Write a mathematical expression that represents the goal you stated in #1 above, using the variables you chose in #2 above. Note that the objective will be an expression rather than an equation.

   Possible answer: Profit from sales is represented by the expression 40x + 65y

5. List the constraints that affect your production of artwork.

   Possible answer:

   - ***Budget constraint: can spend up to $240***
   - ***Time constraint expressed as the total number of art pieces: 20 pieces***
   - ***Costs of supplies: each drawing costs $8 and each mixed-media collage costs $16***
   - ***The lowest value of the number of drawings I can make is 0***
   - ***The lowest value of the number of mixed-media collages I can make is also 0 (I can never make a negative amount of pieces!)***

Step 2: Organize the constraints.

Use the table below to record information about the constraints you identified.

Table of Constraints			
	Drawings *x*	**Mixed-media collages** *y*	**Quantities of Input Available**
Time (number of art pieces)	x	y	*at most 20 art pieces*
Cost of supplies	*$8 each*	*$16 each*	*$240*
Number of drawings			*x must be greater than or equal to 0*
Number of mixed-media collages			*y must be greater than or equal to 0*

Step 3: Work with each constraint.

Your teacher will provide you with graph paper or gridded transparencies on which you will create graphs as you work with each constraint.

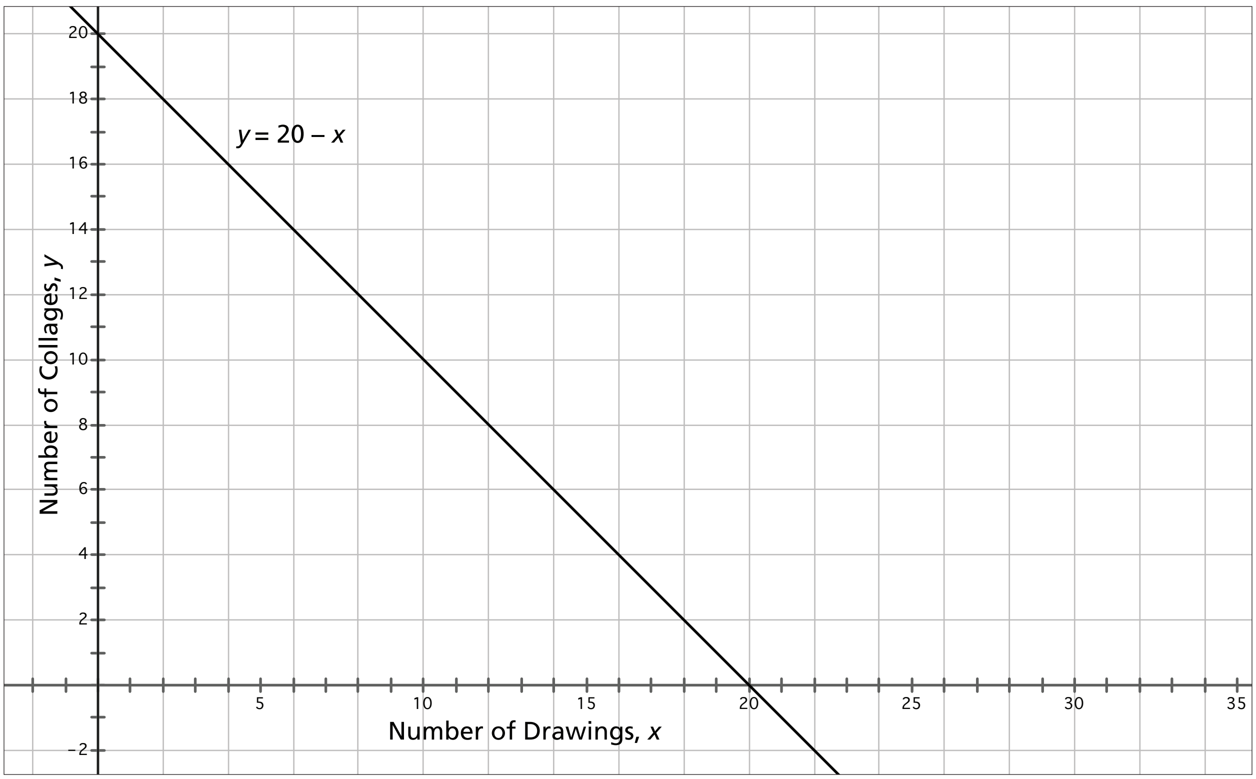
Constraint 1: Time

Plot all of the possible combinations of drawings and mixed-media collages that add up to a total of 20. Label the *x*-axis using your independent variable and the *y*-axis using your dependent variable.

- What kind of function do you obtain? Explain.

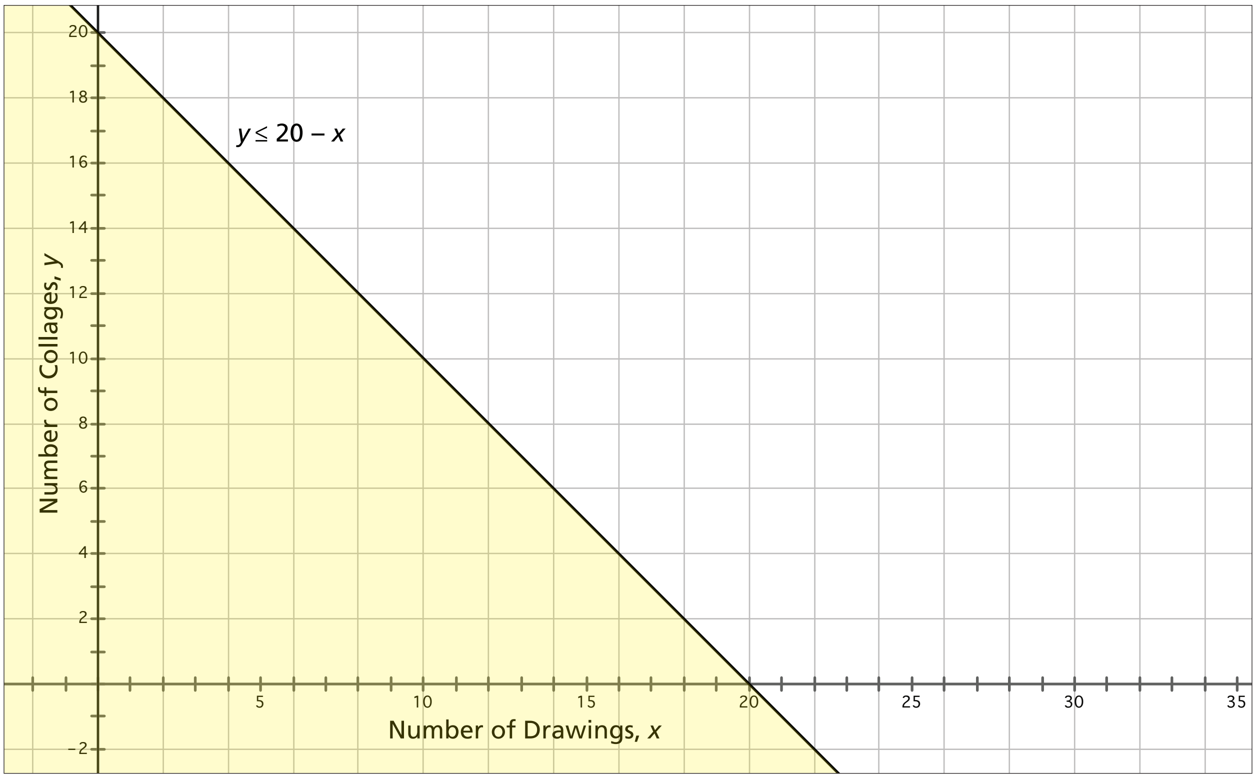
  Answer: I get a linear function because the relationship between the number of drawings and the number of mixed-media collages that I can make is linear. Increasing the number of drawings by 1 always decreases the number of collages I can make by 1. The graph is shown below.

Producing exactly 20 pieces:



- Is it possible to make fewer than 20 pieces? Shade the part of the graph that represents 20 art pieces or fewer.

  Answer:
  Producing 20 art pieces or fewer:



- What inequality could you write to represent this relationship?

  Answer: $x + y \leq 20$

- How could you obtain this inequality using the table of constraints from Step 2 rather than the graph you created?

  Possible answer: Translate the row labeled 'Time' into an inequality.

Constraint 2: Cost of supplies

Follow the steps below to create a graph that represents the financial constraint on supplies.

- Write an expression that represents the money you will spend on supplies if you create *x* drawings.

  Answer: 8x

- Write an expression that represents the money you will spend on supplies if you create *y* mixed-media collages.

  Answer: 16y

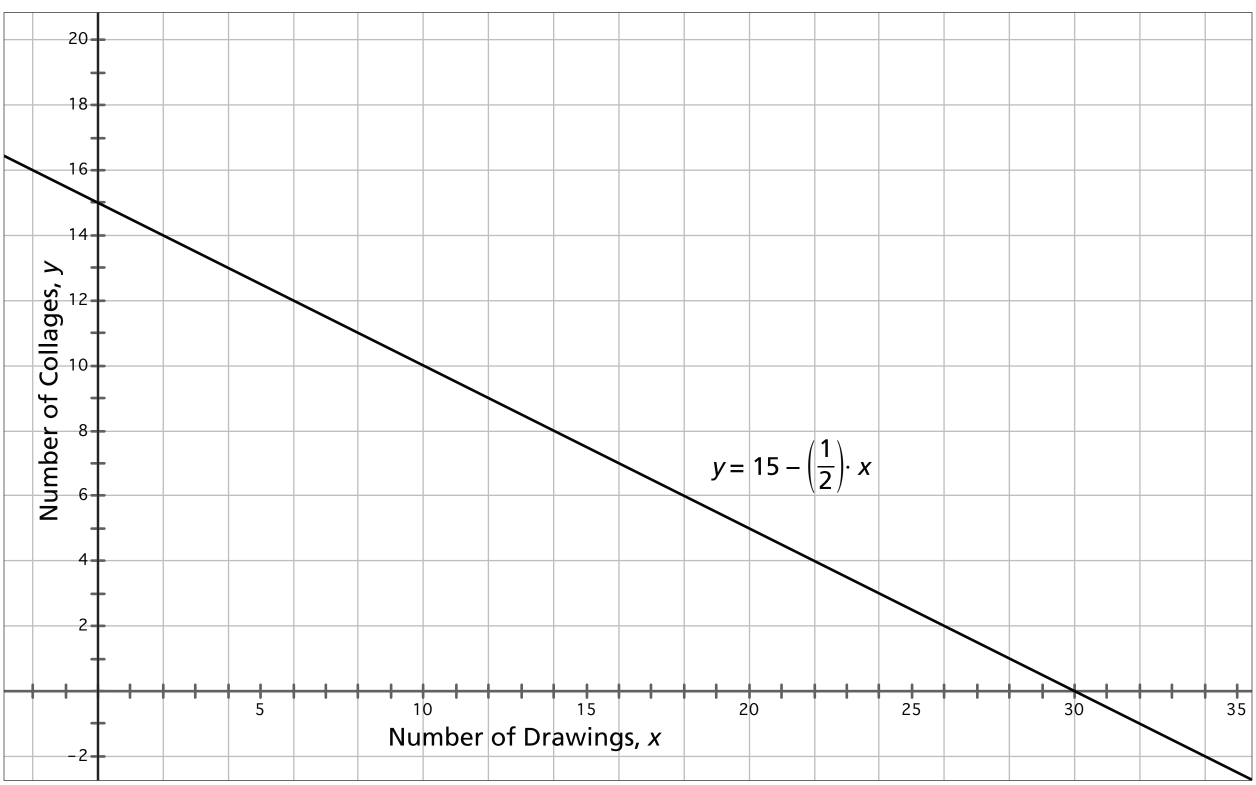
- Write an *equation* that shows that the total amount of money you will spend on supplies when creating *x* drawings and *y* mixed-media collages is *exactly* $240.

  Answer: 8x + 16y = 240

- Explain why the equation you wrote is a linear equation. Graph the equation on a set of axes. (Remember that you can use the *x*- and *y*-intercepts when graphing). Keep the labeling consistent with your previous graph.

  Answer: The equation above can be re-written in slope-intercept form as follows:

  $y = 15 - \frac{1}{2}x$. Interpreting the meaning of the slope tells you that increasing the number of drawings by 2 always decreases the number of collages you can make by 1.



- Write an *inequality* that expresses the financial constraint on supplies.

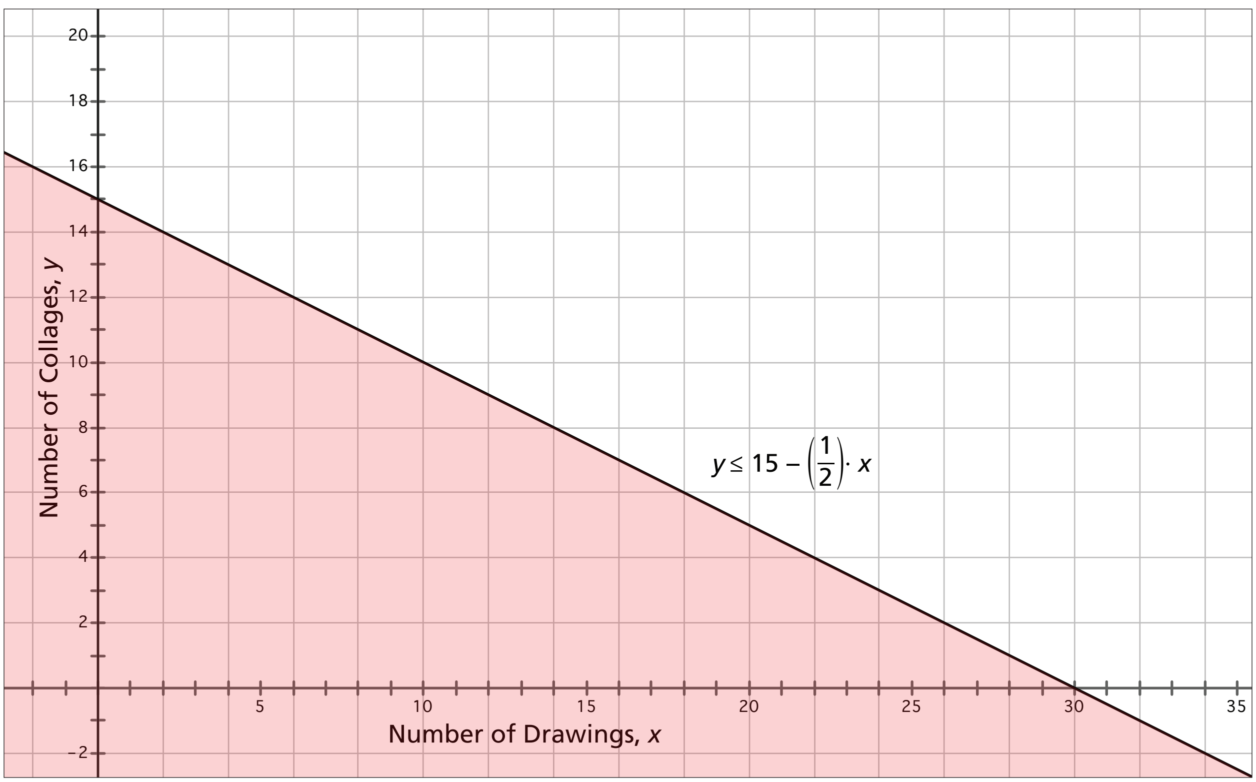
  Answer: *$8x + 16y \le 240$*

- Graph the inequality. The inequality will split the plane into two regions.

  Choose a point in one of the two regions of the plane and test its coordinates in the inequality. If the point satisfies the inequality, shade the part of the graph that includes that point. If the point doesn't satisfy the inequality, choose a point in the other region of the plane to test.

  The part of the graph that satisfies the inequality is the solution set for this constraint. Shade this part of the graph.

  Answer:
  Testing the point (0, 0) in the inequality, $8x + 16y \le 240$ makes this statement true. Shade the part of the graph that includes this test point.



Constraint 3: Number of drawings

Look at your table of constraints. Write an inequality to express the constraint on the number of drawings.

Note that this inequality, as well as the inequality that represents the constraint on the number of collages, can be written as a *one-variable* inequality. Recall that you can also represent one-variable inequalities on the two-dimensional coordinate plane.

Graph this inequality.

Answer:
$x \geq 0$
See graph below.

Constraint 4: Number of mixed-media collages

Write an inequality to express the constraint on the number of collages. Remember that you can write this constraint as a *one-variable* inequality.

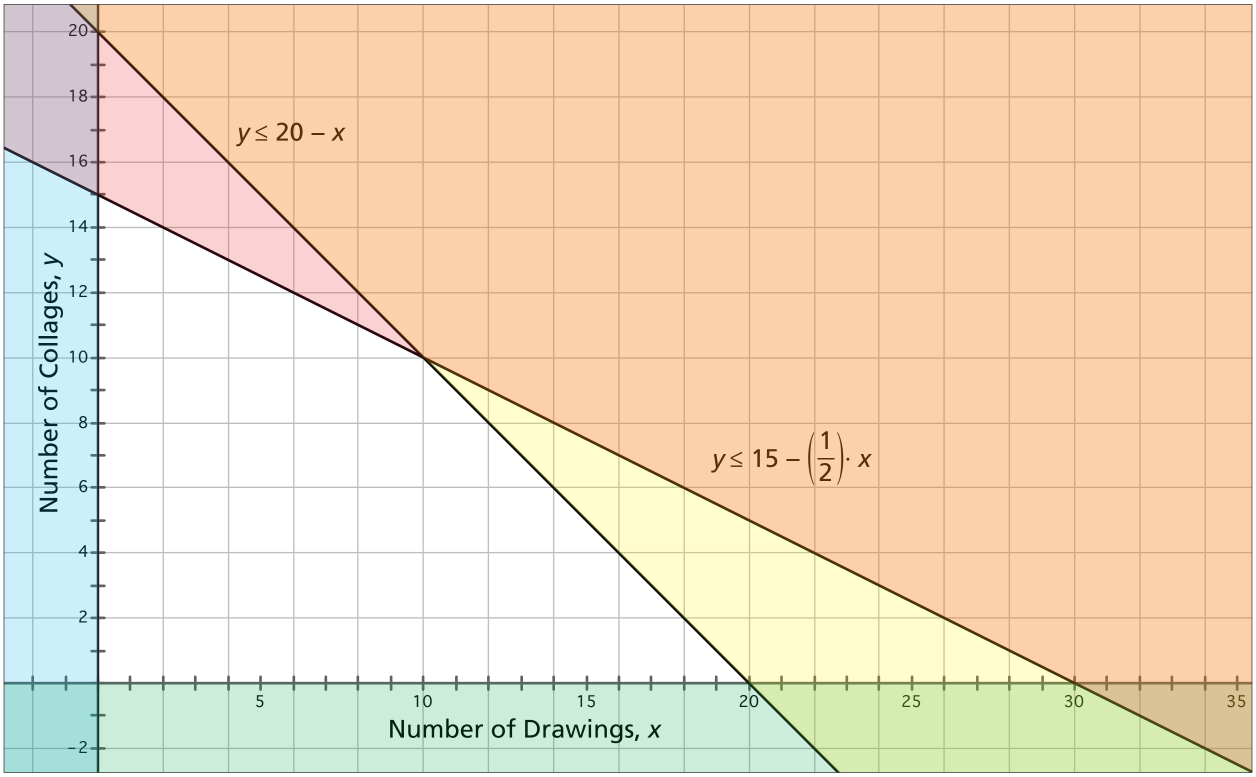
Graph this inequality.

Answer:
$y \geq 0$

Teacher's Notes: Shading Graphs

As shown in the final graph on p. 56, it is sometimes easier to shade the part of the graph that does *not* satisfy an inequality. This is particularly useful when combining the solutions to multiple inequalities on one coordinate plane. Then the solution of the set of inequalities is the portion of the graph that is *not* shaded rather than the portion where all the shading overlaps.

The graph shown was created using *Geometer's Sketchpad*. In this case it was clearer to shade the portions of the coordinate system that do not satisfy the inequalities.



You have created a mathematical model for a linear programming problem. Later in the unit, you will use this model to find the *optimal solution* for Problem B—Profit Maximization.

2A.2: Constraints in Problem A—Media Selection

Students return to Problem A—Media Selection. The class creates a mathematical model for the problem by choosing decision variables, identifying an objective function, and representing the problem's constraints in terms of the decision variables.

1. Determine the decision variables as a class.
Tell students that they are going to create a mathematical model for Problem A, the media selection problem.

Have students refer to their copies of **Handout 2: Problem A—Media Selection**, **Handout 4: Problem A—Media Selection: More Information** and their work for each handout.

Discuss what the decision variables might be for this problem. Remind students that the decision variables determine the expression for the objective function and that the decision variables are in some ways constrained by limits on resources.

As students suggest possible decision variables, ask:

- How do these variables influence the goal or objective of the problem?
- What information given in the problem might set limitations on these variables?

  Answer: *Use* r *and* n *as decision variables.*

  r = *the number of sets of 3 radio spot ads to run during the promotion period*

  n = *the number of newspaper ads to run during the promotion period*

  The two variables r *and* n *determine the objective function because each has the potential to reach a specific number of people. Limitations on these variables include the cost to run the ads and the budget constraints set by the Executive Director and the sponsors of the festival.*

2. Write the objective function in terms of the chosen decision variables.
Display the prompts given below and have students write individual responses in their notebooks. Tell students to refer to their work on Handout 5, in which they calculated the estimated reach for radio and newspaper advertising.

- What effect does the first variable, r, have on media reach? Write a mathematical expression that shows this effect.

  Answer: *3,960*r *represents the number of people reached when* r *sets of 3 radio spot ads are purchased.*

- What effect does the second variable, *n*, have on media reach? Write a mathematical expression that shows this effect.

  Answer: *2,000*n *represents the number of people reached when* n *newspaper ads are purchased.*

- Write a mathematical expression for the objective function of the problem. What are you trying to *optimize*?

  Answer: *I am trying to optimize* reach*, or the number of people that learn about the media festival through promotion efforts and are likely to attend. Represent the objective function with the following expression:*

  *3,960*r + *2,000*n

Ask volunteers to share their results with the class.

Make sure that the class agrees with the chosen decision variables and the objective function determined by those variables.

On the board, write the two decision variables, the quantities they represent, and the objective function.

Ask students:

- What information given in the problem poses constraints on the objective function?

Tell students to refer again to Handout 2, Handout 4, and their responses to the handouts. Remind students that they should try to identify any constraints on the number of radio spot ads to purchase and/or on the number of newspaper ads to run.

Ask students to share their responses and have a volunteer record responses on the board.

Answers: *Constraints include:*

The committee has a total of $4,000 *to use for promotion.*

A quarter-page newspaper ad costs $70*.*

A set of three 30-second radio spot ads costs $200*.*

The entire promotion takes place during the three weeks before the festival. This places a maximum value for each of the two decision variables. Since three weeks is 21 days, the number of newspaper ads has to be less than or equal to 21*, and* the number of sets of three radio spot ads has to be less than or equal to 21*.*

The ratio of the number of days the radio ads run to the number of days the newspaper ads run must be between ½ and 2.

Have students record the information on the board in their notebooks. Tell them that they will use this information to create a mathematical model for the problem.

Teacher's Notes: Writing Constraints Using Decision Variables

When you think about constraints in the problem, you pay attention to how the constraints directly limit the number of radio spot ads or the number of newspaper ads that can be placed. The choices for the decision variables are simplified for the purpose of creating a mathematical model.

Putting boundaries on the problem by assigning two decision variables and writing the constraints in terms of these variables makes it possible to resolve the problem. When it's time to implement the solution to the problem, you can reconsider the limitations.

3. Have students translate constraints into linear inequalities.
Divide the class into groups of three or four. Assign each group one of the five constraints. Give groups 10 minutes to explore ways in which to write their constraints as linear inequalities using the two decision variables.

Teacher's Notes: Managing Groups

If you have more than five groups of students, you can assign the same constraint to more than one group.

You may want to consider grouping the students by ability and assigning the different constraints to these groups accordingly. Specifically, the two non-negativity constraints—$r \geq 0$ and $n \geq 0$—and the time limit constraints are the simplest, the budget constraint is moderately complex, and the restriction on the ratio of the ads is the most complex.

If a group finishes early, have them write one or more of the other constraints as inequalities.

Each group shares its results with the class by doing the following:

- one student writes the inequality(ies) on the board
- another student explains the process used to generate the inequality(ies).

Answers: See mathematical model on pages 60–61.

Teacher's Notes: Table of Constraints for Problem A—Media Selection

If students have difficulty translating the constraints into linear inequalities, work with them to generate a Table of Constraints similar to the table used on Handout 7 for Problem B—Profit Maximization.

Display Table of Constraints: Problem A—Media Selection and fill in the table as a class. Using the table to organize the information in the problem will be helpful later in the problem-solving process when students graph each inequality.

4. Have students write the complete mathematical model for Problem A.
Tell students to write the complete mathematical model in their notebooks. The model should include:

- a description of each decision variable
- a descriptive statement of the objective
- the linear expression that represents the objective (the objective function)
- a short description of each constraint
- a linear inequality that represents each constraint

Tell students that they will also need to include *non-negativity* constraints. Point out that neither the number of radio spot ads nor the number of newspaper ads can have a negative value.

Answer:

Problem A—Media Selection
Mathematical Model

Decision variables:
r = number of sets of 3 radio spot ads to run during the three-week promotion period
n = the number of newspaper ads to run during the promotion period

Objective:
To determine the number of radio spot ads and the number of newspaper ads that maximize media reach subject to specific constraints. Reach is represented by the expression $3{,}960r + 2{,}000n$

Constraints:
Non-negativity: $r \geq 0$, $n \geq 0$

Budget: $200r + 70n \leq 4{,}000$

Time limit for promotion: $r \leq 21$ and $n \leq 21$

Additional restrictions on budget:

$$\frac{1}{2} \le \frac{r}{n} \le 2$$

This inequality can be expressed in two parts:

$$n \ge \frac{1}{2} r \text{ and } n \le 2r$$

2A.3: Partner Work—Problem Formulation

Students work in pairs as they practice formulating linear programming problems.

1. Give pairs example linear programming problems.
Tell students that now that they have created mathematical models for two linear programming problems, Problem A and Problem B, they and their partners are going to work together on another linear programming problem.

Gather the example linear programming problems you selected and distribute one problem to each pair of students. Have pairs work together to:

- identify the decision variables for their problem
- write an expression for the objective function.

Remind students to write the objective function in terms of the decision variables.

Ask volunteers to share their problems with the class and explain how they chose the decision variables, as well as how they came up with the objective function.

2. Have students identify constraints.
Have pairs work together again to create a Table of Constraints to organize the limitations on the resources for their problem. Circulate around the classroom to respond to questions and to check students' work.

Activity 2B: The Feasible Region

Students revisit both linear programming problems, Problem B—Profit Maximization and Problem A—Media Selection, to find the values of the decision variables that satisfy all of the constraints in each problem. Student partners discuss objectives and constraints in their own linear programming problems.

Sequence

2B.1: ***The Complete Graph and the Feasible Region: Problem B—Profit Maximization***	Students work together to find all of the possible solutions, or the feasible region, for maximizing profit in Problem B—Profit Maximization.
2B.2: ***The Complete Graph and the Feasible Region: Problem A—Media Selection***	Students work on their own to find the feasible region, or all the possible solutions for maximizing reach in Problem A— Media Selection.
2B.3: ***Partner Work***	Student pairs formulate objectives and identify constraints in their own linear programming problem.

Understandings

- The feasible region consists of the solutions that satisfy all of the constraints. This is often an infinite set.
- A linear programming problem is infeasible if there are not values of the decision variables that satisfy all of the constraints of the problem.

Materials Needed

- **Handout 8: Problem B—Profit Maximization: The Feasible Region**
- Students' copies of **Handout 7: Problem B—Profit Maximization**
- Students' graphs (created on paper or on transparencies) from **Handout 7: Problem B—Profit Maximization**
- Supplies to create consolidated graphs:
    - Colored dry-erase markers and large sheets of laminated graph paper (one per pair) (if students created their Handout 7 graphs on sheets of graph paper)

- Transparency markers and transparencies with grids (one per pair) (if students created their Handout 7 graphs on transparencies)
- Rulers
- **Handout 9: Problem A—Media Selection: Mathematical Model**
- Optional: Students' copies of **Handout 2: Problem A—Media Selection**
- Optional: Students' copies of **Handout 4: Problem A—Media Selection: More Information**
- Optional: Students' work on Handouts 2 and 4 (from Activities 1A.2 and 1B.1)

2B.1: The Complete Graph and the Feasible Region: Problem B—Profit Maximization

Students work together to find all of the possible solutions to Problem B—Profit Maximization.

1. Distribute Handout 8: Problem B—Profit Maximization: The Feasible Region. Tell students that they will now work on finding all the possible solutions for Problem B—Profit Maximization. Explain that they will use their work on Handout 7 to help them find these solutions.

Review with students the mathematical model for Problem B given on Handout 8.

Have student pairs gather their copies of **Handout 7: Problem B—Profit Maximization** and the graphs they created for Handout 7.

Teacher's Notes: Preparing to Consolidate Graphs

If students created their graphs on graph paper rather than transparencies, give pairs a large sheet of laminated graph paper to use for consolidating their graphs. Provide dry-erase markers in different colors.

If students created their graphs on transparencies, ensure that pairs have one complete set of graphs on different transparencies.

2. Have students combine the constraints for Problem B—Profit Maximization. Explain to students that they will consolidate the graphs of all of the constraints onto one coordinate plane.

Teacher's Notes: Consolidating Graphs

Students working with laminated graph paper: Allow time for students to re-create their graphs using different colors for each constraint. Remind students to pay careful attention to the scaling of the x- and y-axes so that they accommodate the x- and y-intercepts of each of the linear constraints.

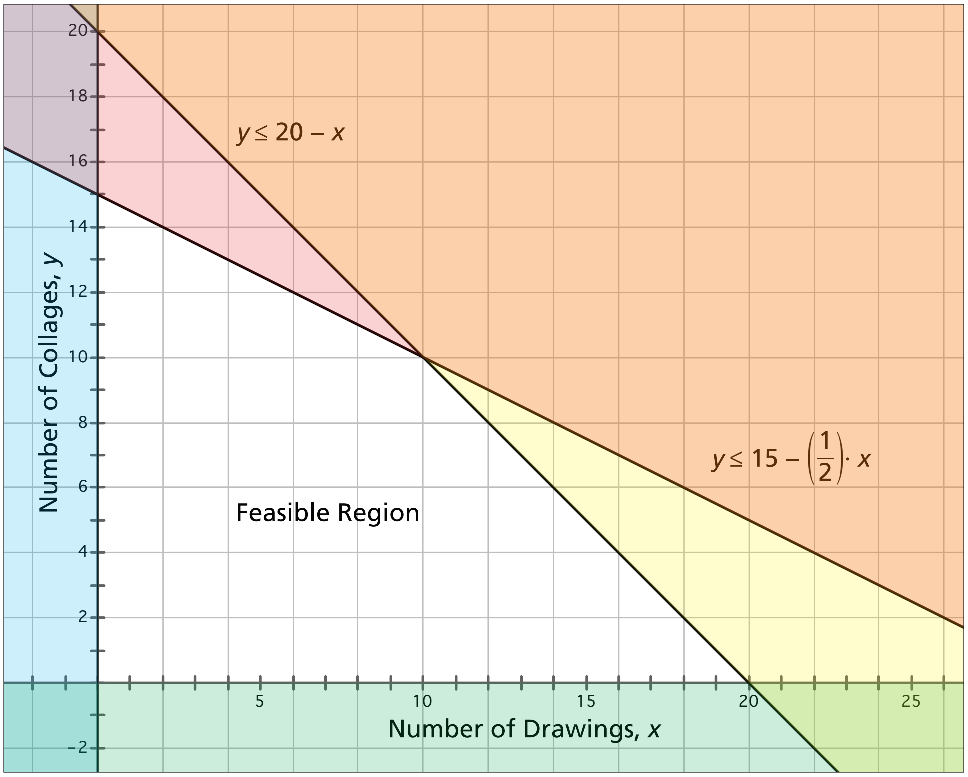
Students working with a set of transparencies: Have students assemble the transparencies one on top of another, lining up the axes. Then have students place a blank gridded transparency on top and create a consolidated graph.

Students use their consolidated graphs in Activity 3A.1 when they find the optimal solution to Problem B.

Tell students to clearly mark the region where all of the constraints are satisfied. Explain that this region of points on the plane is called the *feasible region* and that this region represents the solution set for the problem. It is the region where all constraints are met.

The feasible region for Problem B—Profit Maximization is shown in the graph on page 66. Check that students have identified this region on their laminated graphs or on their overlapping transparencies.

Graph: Feasible Region for Problem B—Profit Maximization



Teacher's Notes: Linear Inequalities with > or <

In Problem B—Profit Maximization, all the constraints are written with the inequality symbols *greater than or equal to*, ≥, and *less than or equal to*, ≤. Discuss with students how they might represent linear inequalities in two variables on the plane when the inequality does *not* include the points on the boundary.

Ask students:

How can you express greater than (>) or less than (<) on the coordinate plane?

Answer: *When the solution of a linear inequality does not include the points on the boundary line, you represent the boundary with a dashed line.*

3. Investigate possible number of solutions to Problem B—Profit Maximization. Ask students to use their consolidated graph to respond to the follow-up questions on **Handout 8: Problem B—Profit Maximization: The Feasible Region**.

> **Note:** You may want to answer a few questions on Handout 8 together as a class and then have students answer the remaining questions on their own.

Discuss students' responses as a class. Make sure that students understand that all points lying within and along the boundaries of the feasible region satisfy the constraints of this linear programming problem.

Display the concepts listed below on the board. Have students refer to their graphs as you discuss these concepts with them:

- Each constraint eliminates part of the plane.
- The *feasible region* is the set of all ordered pairs that satisfy all of the constraints of the linear programming problem. There are an infinite number of solutions contained in this set.
- When a feasible region does not exist, the situation is said to be *infeasible*. In that case, there are no points on the plane that satisfy all of the constraints of the problem.
- Infeasibility is independent of the objective function. It occurs when the constraints are too restrictive, and it may mean that the problem needs to be reformulated.

Discuss with students how to make a change in one or more of the constraints in Problem B—Profit Maximization in order to create an infeasible region.

Teacher's Notes: Optional Extension: Unbounded Region of Feasibility

You may want to also discuss the concept of an *unbounded* solution with students. A solution to a linear programming problem is *unbounded* if the value of the solution can be made infinitely large while still satisfying all of the constraints.

In a profit maximization problem, an unbounded solution would mean that there are no limits to the profit that can be made; in other words, unlimited profit can be achieved. An unbounded solution may mean that the problem has not been well formulated and does not accurately represent a real-world situation.

Handout 8:
Problem B—Profit Maximization: The Feasible Region

You developed a mathematical model for Problem B through your work on **Handout 7: Problem B—Profit Maximization**.

Fill in the model using your work from Handout 7. Then answer the follow-up questions on this handout to work toward finding the solutions to Problem B—Profit Maximization.

Problem B—Profit Maximization
Mathematical Model

Decision variables:

Objective, with Objective Functions:

Constraints:

Additional restrictions:

Follow-up Questions

Use the mathematical model and your completed graph showing all of the constraints to respond to the questions below.

1. Label a few points on the boundary of the feasible region and interpret what these particular points indicate.

   Possible answer:
   The point (0, 14) is on the boundary of the feasible region. This point meets all of the constraints; in other words, it is possible to create 0 drawings and 14 mixed-media collages for the festival.

2. Label a few points on the outside of the feasible region and interpret what these particular points indicate.

   Possible answer:
   The point (16, 6) is outside of the feasible region. It does not satisfy the constraint that the total number of pieces of artwork must be less than or equal to 20. The point (16, 6) represents creating 22 pieces of artwork, 16 drawings and 6 mixed-media collages.

3. Could you create 10 drawings and 4 mixed-media collages while satisfying all of the constraints of the problem? Explain your response.

   Answer:
   Yes. The point (10, 4) falls within the feasible region of the problem.

4. Could you create 4 drawings and 14 collages and satisfy all of the constraints of the problem? Why or why not?

   Answer:
   No. The point (4, 14) falls outside of the feasible region.

5. Could you create 15 drawings and 2 collages and satisfy all of the constraints? If so, what profit would you make from selling this combination of artwork pieces, assuming that all of them can be sold?
   Hint: Use the given values to evaluate the objective function.

   Answer:
   Yes. The point (15, 2) is in the feasible region.
   Profit = $40x + 65y = 40(15) + 65(2) = 600 + 130 = 730$.
   Your profit will be $730 if you create and sell 15 drawings and 2 collages.

6. Which combination will give you a higher profit: 10 drawings and 4 collages or 5 drawings and 5 collages?

   Answer:
   10 drawings and 4 collages gives you a profit of $660.
   40(10) + 65(4) = 400 + 260 = $660

   5 drawings and 5 collages gives you a profit of $525.
   40(5) + 65(5) = 200 + 325 = $525.
   Making and selling 10 drawings and 4 collages is the more profitable combination.

7. Are there other possible combinations of drawings and mixed-media collages that satisfy all of the constraints in the problem? Explain.

   Answer:
   Yes, there are other possible combinations. The feasible region suggests that there are an infinite number of combinations since there are an infinite number of points within this region. However, you would like to sell complete pieces of artwork, not partially-finished pieces. So, other possible combinations are points within the feasible region that have integer values.

2B.2: The Complete Graph and the Feasible Region: Problem A—Media Selection

Students work on their own to find the feasible region, or all the possible solutions for maximizing reach in Problem A—Media Selection.

1. Have students find the feasible region for Problem A—Media Selection. Distribute **Handout 9: Problem A—Media Selection: Mathematical Model**. Tell students that now they are going to find the feasible region for this problem.

Review the mathematical model given on the first page of Handout 9. Have students also refer to **Handout 2: Problem A—Media Selection**, **Handout 4: Problem A—Media Selection: More Information**, and their work on the handouts, if necessary.

Have students work individually on graphing the constraints by referring to the problem-solving approach used in Handouts 7 and 8. While students create the graph of the feasible region, circulate around the class and provide assistance. Encourage students to also ask and answer each other's questions.

If students feel intimidated by all of the information in the mathematical model, provide them with a framework for addressing the problem by posting the following prompts:

- Begin your graph by scaling the *r* and *n* axes so that the *r*- and *n*-intercepts of each constraint are visible.
- Note that the non-negativity constraints imply that you will only need to set up axes for the first quadrant of the coordinate plane. This is the only quadrant for which both *x*- and *y*-values are positive (non-negative).
- Graph one constraint at a time. First graph the linear equation and then determine the area of the plane to shade by testing points in the original inequality.
- Use different colors to shade each constraint. The area where all of the shaded regions intersect is the *feasible region*. This region contains all of the solutions to the problem.
- In the next part of the unit, you will use the objective function to determine the *best* solution.

Answer:

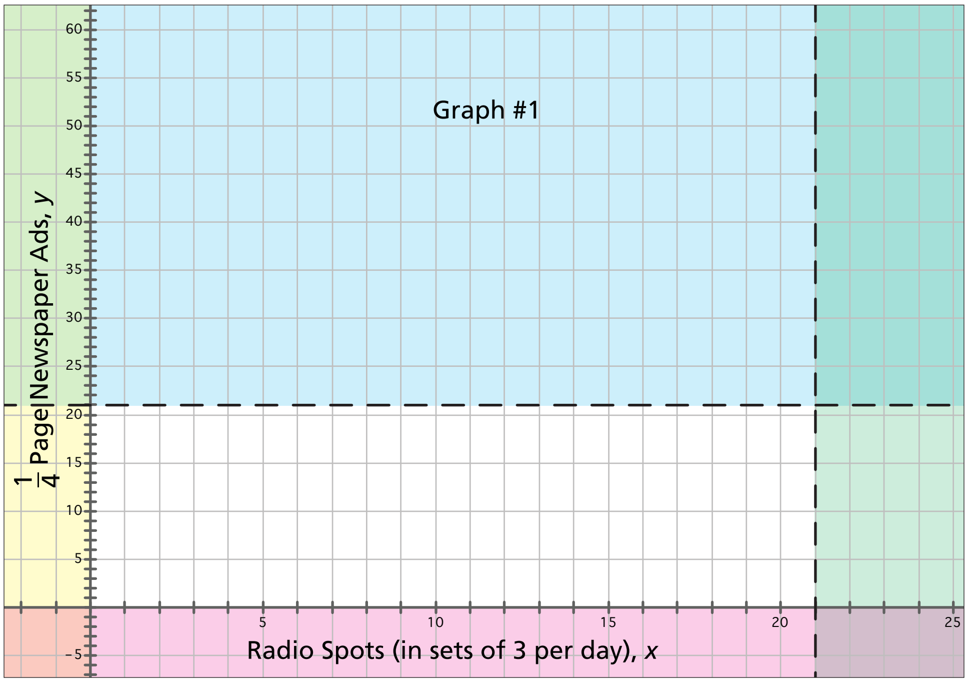
The constraints and the feasible region for Problem A—Media Selection are shown below.

***Graph 1** shows the non-negativity constraints and the time constraint as it affects the two decision variables. Rather than shading the portion of the plane that satisfies each inequality, the graph below is created by shading the parts of the plane that do not satisfy each inequality. In this way, the feasible region will be the portion of the plane that is* not *shaded.*

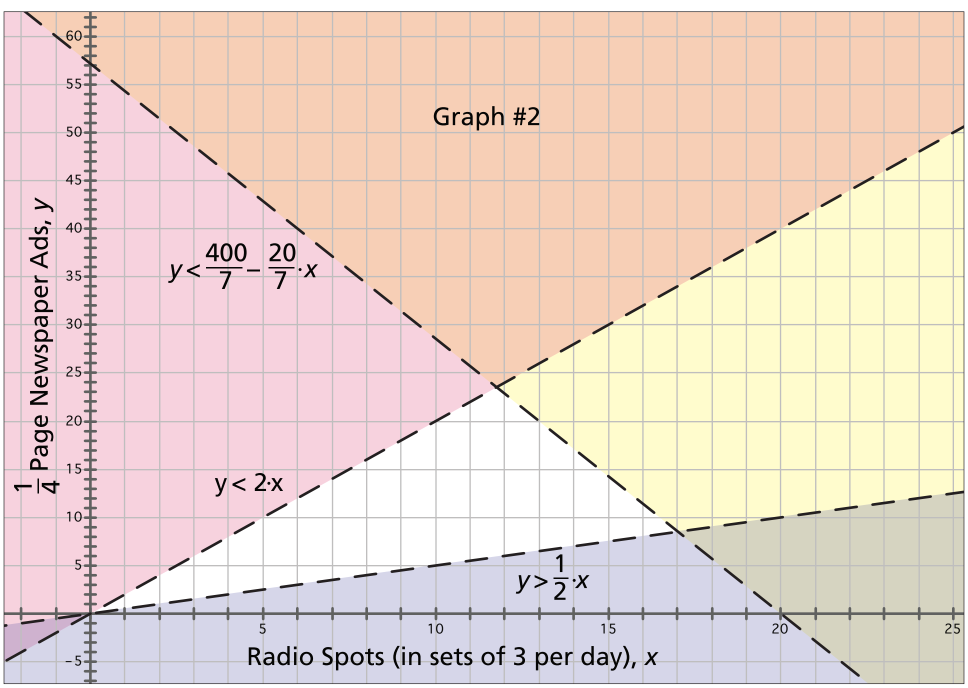
__Graph 2__ shows the budgetary constraints, including those imposed by the sponsors of the festival. A shading strategy similar to that in Graph 1 is used.

__Graph 3__ combines all of the constraints to show the feasible region. The feasible region is the non-shaded portion of the graph.

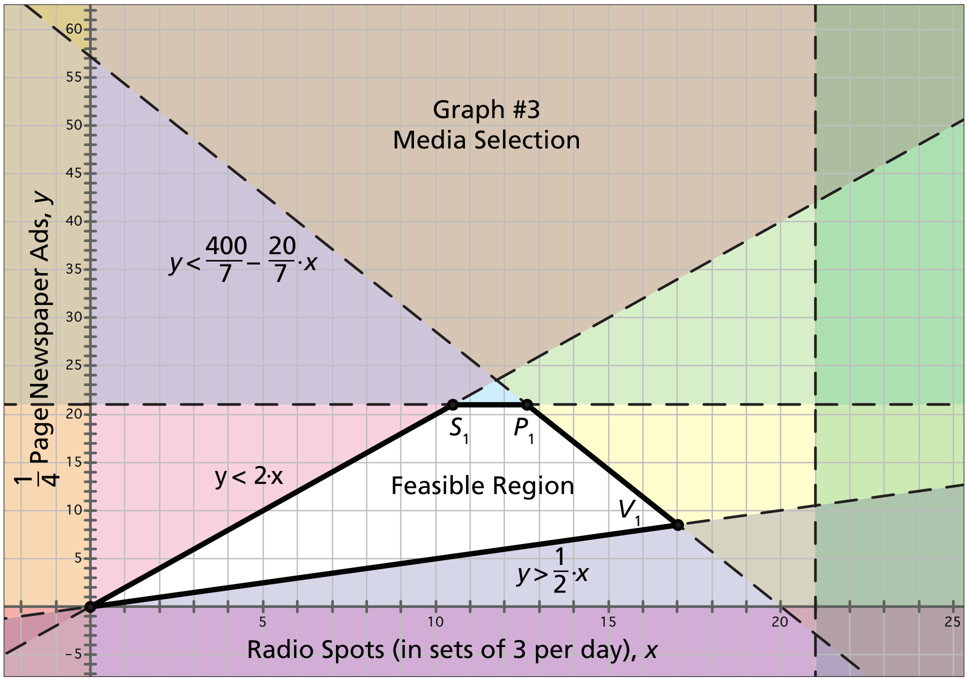
Graph 1: Non-negativity and time constraints for Problem A—Media Selection



Graph 2: Budget constraints for Problem A—Media Selection



Graph 3: Feasible region for Problem A—Media Selection



2. Collect student work and provide feedback.
Review students' work and have them revise the graphs as needed.

Note: Students will refer to the graph they create in this activity when they determine the optimal solution for Problem A in Part 3 of the unit.

3. Have students interpret their graphs.
Have students use their graphs to answer the questions on **Handout 9: Problem A—Media Selection: Mathematical Model**. Collect students' work.

Handout 9:
Problem A—Media Selection: Mathematical Model

The mathematical model for Problem A is given below. Find all of the possible solutions, or the feasible region, for the problem.

First, use the model to create a graph. Then answer the follow-up questions on this handout. Your teacher will check your work.

Problem A—Media Selection

Mathematical Model

Decision variables:

r = number of sets of 3 radio spot ads to run during the three-week promotion period
n = the number of newspaper ads to run during the promotion period

Objective:

To determine the number of radio spot ads and the number of newspaper ads that maximize media reach subject to specific constraints.
Reach is represented by the expression $3{,}960r + 2{,}000n$

Constraints:

Non-negativity: $r \geq 0$, $n \geq 0$

Budget: $200r + 70n \leq 4{,}000$

Time limit for promotion: $r \leq 21$ and $n \leq 21$

Additional restrictions on budget:

$$\frac{1}{2} \leq \frac{r}{n} \leq 2$$

This inequality can be expressed in two parts:

$$n \geq \frac{1}{2}r \quad \text{and} \quad n \leq 2r$$

Follow-up Questions

1. Label a few points on the boundary of the feasible region you found and interpret what these particular points indicate.

   Possible answer:
   The points (0, 0) and (12, 21) lie on the boundary of the feasible region. These points meet all of the constraints of Problem A—Media Selection. It is possible to place 12 sets of radio spot ads and 21 newspaper ads and satisfy all of the problem's constraints.

2. Label a few points on the outside of the feasible region and interpret what these particular points indicate.

   Possible answer:
   The point (4, 21) is not within the feasible region of the problem. This means that the point does not meet one or more constraints. This particular point does not satisfy the sponsor's constraint that $n \leq 2r$.

3. Could you broadcast 8 sets of 3 radio spot ads and purchase 10 newspaper ads while satisfying all of the constraints of the problem? Explain.

   Answer:
   Yes. The point (8, 10) falls within the feasible region of the problem.

4. Could you place 10 sets of radio spot ads and 10 newspaper ads and still meet all of the constraints? If so, how many people will you reach?
   Hint: Use the given values to evaluate the objective function.

   Answer:
   Yes. The point (10, 10) is in the feasible region.
   Reach = $3{,}960r + 2{,}000n = 3{,}960(10) + 2{,}000(10) = 39{,}600 + 20{,}000 = 59{,}000$ people.

5. Which combination will reach more people: 10 sets of radio spot ads and 10 newspaper ads or 11 sets of radio spot ads and 9 newspaper ads? Why does this make sense given the context of this problem?

   Answer:
   When $r = 10$ and $n = 10$, 59,000 people who plan to attend the festival can be reached.
   For $r = 11$ and $n = 9$, find reach by evaluating the objective function.
   Reach = $3{,}960(11) + 2{,}000(9) = 43{,}560 + 18{,}000 = 61{,}560$.

   It makes sense that the point (11, 9) yields a higher reach because an increase of one set of radio spot ads reaches more people than are reached by an increase of one newspaper ad.

6. Are there other possible combinations of radio spots and newspaper ads that satisfy all of the constraints?

   Answer:

   Yes, there are other possible combinations. The feasible region suggests that there are an infinite number of combinations since there are an infinite number of points within this region. However, in this case, the solutions would need to have integer values to fit the constraints of the problem. If it were possible to have fewer and shorter radio ads in one day, and newspaper ads that are smaller than one-quarter page, other non-integer solutions would also be possible.

2B.3: Partner Work

Students meet with their partners to work on their own linear programming problems.

1. Have pairs generate their own linear programming problems.
Explain to students that now they will work with their partner on their own linear programming problems.

Have students return to the brainstorming notes they recorded in their notebooks during Activity 1C.2. Have them review the objectives they formulated for their problems.

Explain that now that students have worked with constraints in linear programming problems, they can further expand their original ideas for their own problems.

Have student pairs write their word problems and include constraints, or limitations on resources, in the problems. Provide students with the guidelines below:

- Describe the objective of the problem. Check that it is dependent on two quantities that can change, your decision variables.
- Write one or two sentences describing a relationship between the two decision variables. How are they dependent on one another?
- Determine resource limitations that have an effect on the two decision variables. Resource limitations, or constraints, include budget, time, or materials available.

Teacher's Notes:
Students' Linear Programming Problems and *Unit 7: Art Show!*

If students are preparing to stage the art show in *Foundations in Visual Arts, Unit 7: Art Show!*, you may want to have them generate their linear programming problems based on the ideas and materials they are using for the show.

For example, students might formulate a problem about minimizing the cost of producing various quantities of two different kinds of artwork. As another option, students might formulate a problem similar to that of Problem A—Media Selection by selecting two media vehicles they are using to promote the art show.

2. Have pairs share their problems with the class.
Ask volunteers to display their work and point out their problem's objective, decision variables, and constraints.

Teacher's Notes: Alternatives—Partner Pairs and a Gallery Walk

Alternatively, you can have student pairs exchange their problems with another pair and discuss each other's work.

If time permits, you might also consider a gallery walk in which students display their problems on chart paper and walk around the classroom to review one another's work. Students can use sticky notes to comment or ask questions about the work.

Part 3: Getting to a Solution

Students determine how they can choose the best solution within the feasible region of a linear programming problem.

First, students use the objective function in Problem B—Profit Maximization to find the combination of drawings and collages that maximizes profit. Students then use the objective function in Problem A—Media Selection to find the mix of media vehicles that reaches the greatest number of people.

Finally, students examine whether it makes sense to implement these optimal solutions.

Length

3 50-minute sessions

Advance Preparation

- Before Activity 3A.2, decide whether to review with your students how to solve systems of linear equations. Students will need to know how to solve them in order to find the solution for Problem A—Media Selection. You can use **Appendix C: Solving Systems of Linear Equations** for a review.

Activity 3A: How Can You Obtain the Best Solution?

Students find the locations of the points in the feasible region that optimize the objective function. They use these points to obtain the optimal solution to each linear programming problem.

Sequence

3A.1: Optimal Solution: Problem B—Profit Maximization	Students find the optimal solution to Problem B—Profit Maximization. Students see that the optimal solution to a linear programming problem occurs at one or more corner points of the feasible region.
3A.2: Optimal Solution: Problem A—Media Selection	Students find the optimal solution to Problem A—Media Selection. They identify the corner points of the feasible region, find the coordinates of the point that maximizes reach, and determine whether the optimal solution makes sense in the real world.

Understandings

- In linear programming, the optimal solution(s) occur at one or more corner points of the feasible region of the problem.
- When applying a mathematical model to a real-world problem, the solutions need to be checked for viability.

Materials Needed

- Students' copies of **Handout 8: Problem B—Profit Maximization: The Feasible Region**
- Students' consolidated graphs (created on laminated graph paper or transparencies) of the feasible region for Problem B (from Activity 2B.1)
- **Handout 10: Problem B—Profit Maximization: Optimal Solution**
- Transparencies with grids
- Transparency markers
- Rulers
- Optional: Laminated graph paper or chart paper with grids
- Optional: Cardboard strips
- Students' copies of **Handout 9: Problem A—Media Selection: Mathematical Model**
- Students' graphs of the feasible region for Problem A (from Activity 2B.2)

3A.1: Optimal Solution: Problem B—Profit Maximization

Students find an efficient approach for determining the optimal solution in the feasible region of Problem B—Profit Maximization.

1. Have students revisit their work on Handout 8.
Explain that students will use their work on **Handout 8: Problem B—Profit Maximization: The Feasible Region** to find the optimal solution(s) to Problem B.

Have student pairs gather their copies of Handout 8 and their consolidated graphs that show the feasible region for Problem B.

2. Distribute Handout 10: Problem B—Profit Maximization: Optimal Solution.
Tell students that Handout 10 will help them develop an efficient way of searching the feasible region for the best solution to the problem.

Distribute gridded transparencies and markers. Work together as a class on Problem 1. Have student pairs create their graphs on one transparency.

Make sure students understand that the points on each line they graph represent a combination of artworks that yield a profit of $0 and $100, respectively.

Note: When students create their graphs, have them use the same scale as they used on their graphs of the feasible region. They can then overlay the graph of the profit lines on the graph of the feasible region in Problem 2.

Have students work with their partners to complete Problem 2 on Handout 10. Circulate and provide support as needed.

3. Present the Fundamental Theorem of Linear Programming.
Discuss students' responses to Problem 2. Have volunteers share their conjectures.

Present the Fundamental Theorem of Linear Programming and have students note it on Handout 10.

Teacher's Notes: The Fundamental Theorem of Linear Programming

If there is a solution to a linear programming problem, then the solution occurs at a corner point of the feasible region or on a line segment between two corner points. (A corner point is a vertex of the feasible region.)

Have students apply the theorem to Problem B—Profit Maximization. Use the discussion points below to further engage students in interpreting the theorem:

- Points that have the same value of the objective function lie on a line.
- All of the constant profit lines are parallel.
- The value of the objective function is greater for the constant profit lines toward the upper right of the coordinate system.
- As constant profit lines move from the lower left to the upper right of the coordinate system, the last points of the feasible region that the objective function passes through are corner points.
- Because linear equations define the boundaries of the feasible region, the feasible region is a *convex set*. This necessarily means that the objective function bumps into the feasible region at a corner point or at an infinite set of points all of which have the same value of the objective function.
- If two feasible corner points have the same value of the objective function, then all of the points on the line segment joining the two corner points have the same optimum value.

Teacher's Notes: Optional Extension: Exploring the Fundamental Theorem of Linear Programming

You can have students do further work with the Fundamental Theorem of Linear Programming. Give pairs of students an example linear programming problem. (See *Media & Resources*, under Activity 1C.2.) Have them follow the steps below:

- Write the objective function in words and identify the decision variables for the problem.
- Translate the objective function into an expression in terms of the decision variables.
- Represent constraints using linear inequalities.
- Create a graph of the feasible region on laminated graph paper or gridded chart paper. Use cardboard strips for each constraint and paste the constraint lines on the graph.
- Use one cardboard strip to represent the objective function.
- Move the objective function along the surface of the feasible region by assigning to it different values, starting with 0. (Depending on the problem, this represents zero profit, zero reach, zero revenue, etc.)
- Approximate the point on the feasible region that yields the optimum value of the objective function.
- Present the problem to the class, explaining the rationale for the solution process.

4. Have students find the optimal solution to Problem B—Profit Maximization. Have students use their graphs of the feasible region for Problem B to find the corner points. Tell students that for this problem they can read the coordinates of the corner points directly from the graph.

Note: The corner points are located on the x-axis, the y-axis, at the origin, and on an identifiable lattice point on the coordinate plane. See the graph on page 83.

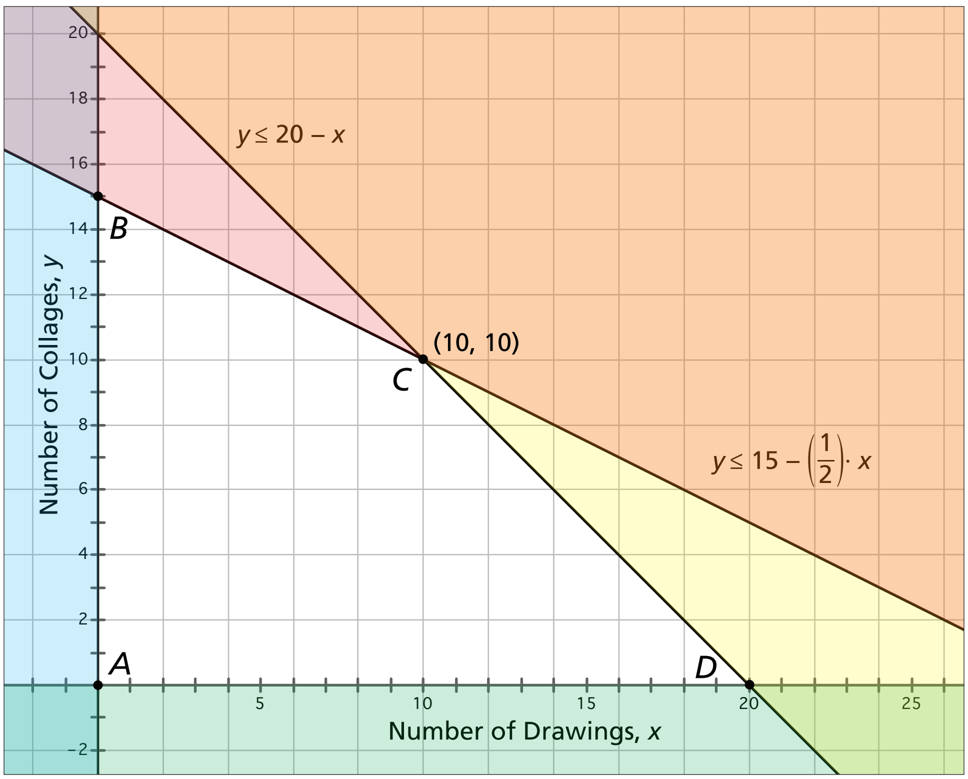
Point out to students that in other linear programming problems they may need to solve systems of linear equations in order to find the coordinates of the corner points.

Teacher's Notes: Systems of Linear Equations

Students will need to solve systems of linear equations to find the solution for Problem A—Media Selection in Activity 3A.2. If your students need a review of the process, you can use **Appendix C: Solving Systems of Linear Equations**.

The graph below shows the corner points (labeled A, B, C, and D) for Problem B—Profit Maximization.

Graph of Problem B—Profit Maximization with corner points identified



Tell students that now that they know the coordinates of the corner points, they can solve Problem B. Explain that they use the coordinates of the corner points to evaluate the objective function.

Display a table similar to the one on page 84, but show only the column headings. Have students copy and complete the table.

Corner Points	Number of drawings, x	Number of mixed-media collages, y	Profit 40x + 65y
(0, 0)	0	0	0
(20, 0)	20	0	40 × 20 = $800
(0, 15)	0	15	65 × 15 = $975
(10, 10)	10	10	(40 × 10) + (65 × 10) 400 + 650 = $1,050

Ask students to state the solution to Problem B. Stress the importance of stating the solution accurately. Tell students their solution should:

- give the coordinates of the corner point that yields maximum profit
- tell what the coordinates represent
- give the value of the maximum profit within the problem's constraints

Students should see that the corner point (10, 10) yields maximum profit. The solution to Problem B can be stated as follows:

- Producing and selling 10 drawings and 10 mixed-media collages yields a maximum profit of $1,050.

Remind students that, as a final step, they need to check whether the solution makes sense in the real world. Ask:

- Is it feasible to create and sell the number of drawings and mixed-media collages that yield the maximum profit?

Handout 10:
Problem B—Profit Maximization: Optimal Solution

You are going to work toward finding the optimal solution for Problem B—Profit Maximization. Use your mathematical model for Problem B, along with the graph you created that shows the feasible region for Problem B.

Your teacher will help you and your partner complete Problem 1. Work together with your partner to complete Problem 2.

1. Recall that the objective function for Problem B is an expression that represents maximizing profit: $40x + 65y$.

   Set the objective function for Problem B equal to 0. Solve for *y*. Graph this linear equation.

   - What is the slope of the line?
   - What is the *y*-intercept?
   - What does any point on this line represent?

     Answers:
     Profit = 40x + 65y
     Setting profit equal to 0 yields the equation 40x + 65y = 0.
     Solving for y gives the standard equation of the line:

     $$y = \frac{40}{65}x$$

     The slope of this line is $-\frac{40}{65}$ ***and the y-intercept is 0. Points that lie on this line will yield 0 profit.***

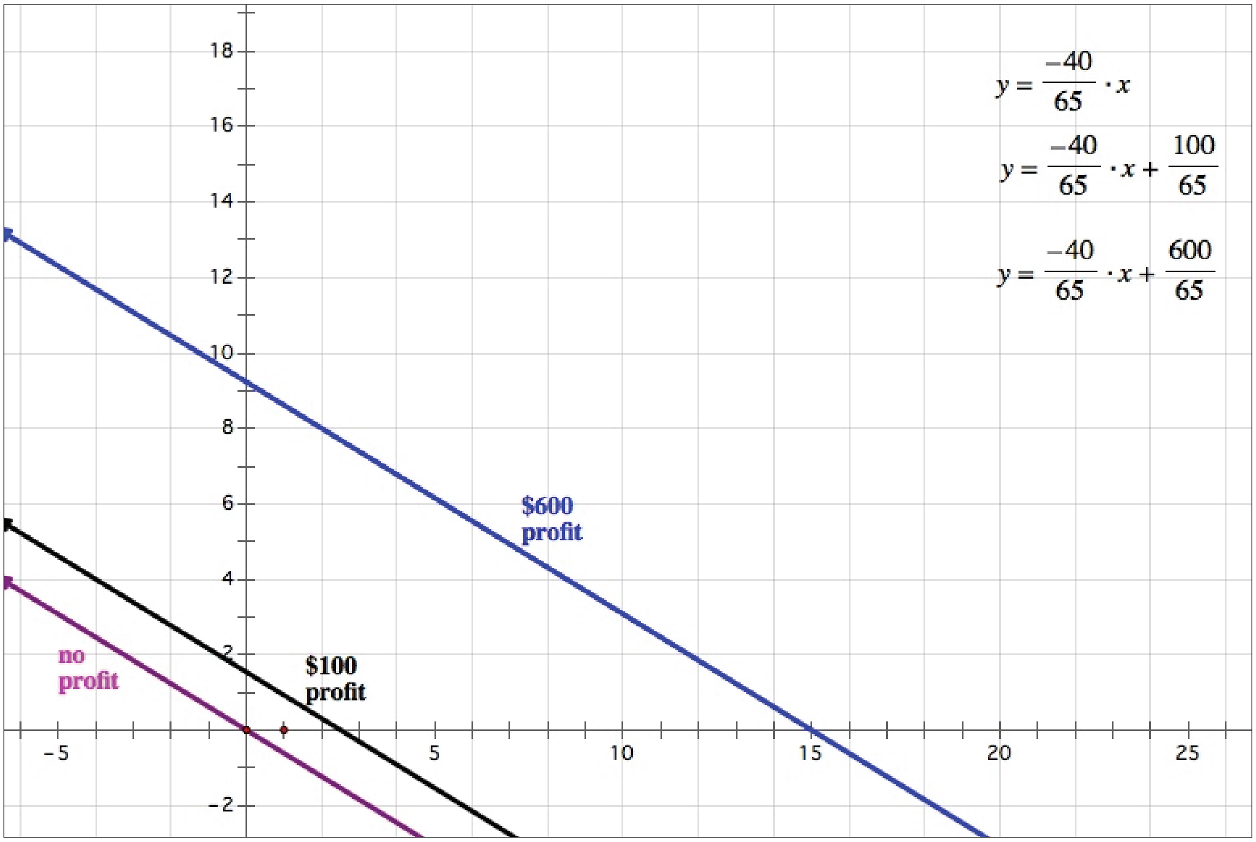
   Now set the objective function for Problem B equal to $100. Solve for *y* once again. Graph this linear equation.

   - What is the slope of this second line?
   - What is the *y*-intercept?
   - What does any point on this line represent?

     Answers:
     Setting profit equal to $100 yields the equation 40x + 65y = 100.
     Solving for y gives the standard equation of the line:

     $$y = -\frac{40}{65}x + \frac{100}{65}$$

     The slope of this line is $-\frac{40}{65}$ ***and the y-intercept is*** $\frac{100}{65}$***. Points that lie on this line represent combinations of drawings and mixed-media collages that yield a profit of $100.***
     The graph below shows the two profit lines, along with a third profit line for Problem B: no profit, $100 profit, and $600 profit.



2. Continue to increase the value of the profit by setting the objective function for Problem B at amounts greater than $100. Do this for at least three new profit values, in increasing order.

   Graph the linear equations. Overlay your graph on the graph of the feasible region for Problem B.

   Use your graphs to answer the questions below:

   - The lines you graphed are called *constant profit lines*. What is the relationship among these lines? How are they similar? How are they different?

     Answer: The constant profit lines are always parallel to each other. They have the same slope and, in this problem, increasing* y*-intercepts.

   - An infinite number of points lie on each constant profit line. How can you determine which points satisfy the constraints of Problem B? What do these points represent?

     Answer: The points on the constant profit lines that satisfy the constraints of Problem B are the points that are also within the feasible region. These points represent the combinations of drawings and mixed-media collages that satisfy the constraints and yield a specified profit (the value of the* y*-intercept of the profit line).

- Remember that you are trying to find the optimal solution to Problem B—Profit Maximization. This solution makes the value of the objective function as great as possible (to maximize profit) while still satisfying the constraints of the problem.

  Use your graphs to help you visualize what happens as profit increases. Think about the location of the constant profit lines and the location of the points on those lines that meet the problem's constraints. Then write a conjecture about the location of the optimal solution to Problem B.

  Answer: As the value of the objective function increases, the constant profit lines move toward the upper right of the coordinate system. For Problem B, the last point in the feasible region that the objective function passes through as it continues to increase occurs at a* corner point *of the region, or at the intersection point of two or more linear constraints.

The Fundamental Theorem of Linear Programming

Answer the following question: What is the Fundamental Theorem of Linear Programming?

Answer:
If there is a solution to a linear programming problem, then the solution occurs at a corner point of the feasible region or on a line segment between two corner points. (A corner point is a vertex of the feasible region.)

3A.2: Optimal Solution: Problem A—Media Selection

Students find the optimal solution to Problem A—Media Selection. They identify the corner points of the feasible region, find the coordinates of the point that maximizes reach, and determine whether the optimal solution makes sense in the real world.

1. Have students revisit their work on Problem A—Media Selection.
Tell students to gather their copies of **Handout 9: Problem A—Media Selection** and their graphs that show the feasible region for Problem A. Explain that they will use their work to find the optimal solution(s) to Problem A.

2. Have students find the solution to Problem A—Media Selection.

> **Note:** Students need to solve systems of linear equations to find the coordinates of the corner points in Problem A. If you want to review how to solve systems of linear equations, you can use **Appendix C: Solving Systems of Linear Equations**.

Tell students you will provide them with a list of instructions they can use to solve Problem A.

Distribute transparencies and markers. Remind students that their graphs should use the same scale as they used on their graphs of the feasible region for Problem A.

> **Teacher's Notes: Scaffolding the Solution Process for Problem A**
>
> Depending on your students' abilities, you can revise the list below by adding to or removing details from the instruction.
>
> Another option is to create an abridged version of the list so that students can apply what they've learned throughout the unit. Students can follow the same problem-solving process for Problem A—Media Selection as they did for Problem B—Profit Maximization.

Display the list shown below. Circulate as students work and provide assistance as needed.

- Find the corner points of the feasible region of Problem A—Media Selection. For some points, you'll need to solve systems of two linear equations.
- Set the objective function equal to 0. Write the linear equation in standard form by solving for *n* (the number of one-quarter page news ads) in terms of *r* (the number of sets of three radio spot ads). Graph the linear equation and identify the slope of the line.

- Overlay the line graph on the graph of the feasible region for Problem A.
- Increase the value of the objective function to 100. Rewrite the equation in standard form and graph the line.
- Continue to increase the value of the objective function. Note the slope of the lines. Overlay the graph with these lines on the graph of the feasible region.
- Estimate the location of the point or points that will maximize reach.
- Create a table similar to the one below. Evaluate the objective function at each corner point of the feasible region.

Corner Points	**Number of sets of radio spot ads, *r***	**Number of news ads, *n***	**Reach** $3{,}960r + 2{,}000n$
(0, 0)	0	0	0

- Solve the problem by identifying the corner point of the feasible region that maximizes reach.
- Determine whether your solution to the problem makes sense in the real world. What does the solution tell you about your media-planning project?
- State the solution to the problem by interpreting the value of the coordinate points that optimize the objective function and by finding the reach of this media plan.

Teacher's Notes: Solution to Problem A—Media Selection

The solution to Problem A—Media Selection follows.

The coordinate points of the feasible region for this problem, as shown in Graph #3, on p. 72 of this unit, are as follows:

S_1 (10.50, 21)

P_1 (12.65, 21)

V_1 (17.02, 8.51)

O (0,0)

The value of the objective function at S_1 is: 3,960 (10.50) + 2,000 (21) = 83,580 people.

The value of the objective function at P_1 is: 3,960 (12.65) + 2,000 (21) = 92,094 people.

The value of the objective function at V_1 is: 3,960 (17.02) + 2,000 (8.51) = 84,419 people.

The value of the objective function at O is 0 people.

Thus, the maximum reach of 92,094 people occurs at P_1 when 12.65 sets of radio ads and 21 quarter-page news ads are placed.

This solution, however, is not practical in the context given in the problem. Therefore, one must investigate the points closest to the optimal solution with integer values. In fact, if the solutions for the corner points of the feasible region were close enough, points close to each corner should be investigated. However, the points close to P_1 (the optimal point) with integer coefficients have solutions with reaches above the other corner points of the feasible region.

The points within the feasible region closest to P_1 that have integer coordinates are (12, 21), and (13, 20).

The value of the objective function at (12, 21) is: 3,960 (12) + 2,000 (21) = 89,520 people.

The value of the objective function at (13, 20) is: 3,960 (13) + 2,000 (20) = 91,480 people.

Thus the maximum reach that can practically be obtained under the given constraints of the problem as it was framed is 91,480 people, which occurs when 13 sets of radio ads and 20 quarter-page news ads are placed.

You may want to discuss with students ways in which to interpret non-integer values of the decision variables as they come up in the solution to the problem, such as suggesting that the committee could investigate the cost and reach of placing fewer or shorter radio spot ads, or smaller newspaper ads.

Discuss with students how the various constraints on Problem A influenced their solution. For example, you might ask students to change one constraint in the original problem and to explore the implications of such a change.

Activity 3B: Completing the Unit Portfolio

Students assemble their portfolios and write a reflection about their work in the unit.

Materials Needed

- Students' copies of Handouts 1–10
- Student's work and graphs related to the handouts
- Optional: Computers (for writing unit reflection)

1. Have students gather their work from throughout the unit.
Have students review **Handout 6: Assembling Your Portfolio** and check to make sure their portfolio is complete.

Have students complete the Students Comments portion of **Assessment Checklist: Unit Portfolio**. Collect students' assessments.

2. Give students time to respond to the unit's framing questions.
Tell students to draft responses to the framing questions shown on Handout 6 and share them with a partner. Have partners revise their responses based on feedback. Tell students to focus on the organization of the responses as well as on the content that directly addresses the framing questions.

3. Ask students to respond to a few final prompts.
Give students time to respond to the following prompts. Then ask students to share their responses with the whole class.

- Something I found challenging in this unit was . . .
- Something that surprised me was . . .
- Something that I would like to explore further is . . .

4. Collect student portfolios in order to assess their learning.

Appendix A: More About Functions and Constant Rates of Change

Use the information below to review functions with students.

Display the equation below:

$$y = 2x + 100$$

Ask students to describe a situation that this equation might represent. For example, the equation can represent the total amount of money a student saves: the initial deposit is $100 and the student saves $2 each week.

Use the example to review the definition of a function and the meaning of an independent and a dependent variable. Ask:

- In the example, what does *x* represent? Why is *x* called a variable?

  Answer: *In the example,* x *represents the number of weeks after the initial deposit. When* x = 0*, the student has $100. At the end of the first week, the student has saved $2, and the total amount is $102. x is a variable because it represents the number of weeks and the number of weeks changes.*

- What does *y* represent? Why is *y* called a variable?

  Answer: *In the example,* y *represents the total amount of money the student has saved after* x *weeks.* y *is a variable because its value changes depending on the number of weeks elapsed.*

- Does it make sense to say "*y* is a function of *x*"? Why or why not?

  Answer: *Yes,* y *changes as* x *changes. The value of* y *depends on the value of* x.

Teacher's Notes: Functions

Students have probably seen and used functions and their representations in previous math classes. You may wish to explain to students that saying that '*y* is a function of *x*', means that a change in *x* produces a change in *y*.

In the example, *y* is the total amount of money the student has at the end of week *x*. (You assume that the student has $100 at the end of week 0, and the student has $102 at the end of week 1).

You might remind students of the more formal mathematical definitions.

Function: A function is a rule that maps each element in one set to exactly one element in a second set.

Independent Variable: A variable whose value *determines* the value of another variable.

Dependent Variable: A variable whose value *is determined* by the value of another variable.

- Is it possible to say that "x is a function of y"? If possible, rewrite the equation above to match this statement (i.e., solve for x).

  Answer: *Yes. Each distinct dollar amount can be mapped to exactly one week. To solve the equation for* x*:*

  $$x = \frac{y}{2} - 50$$

  Note: You can explain to students that functions that exhibit this property (each input can be mapped to exactly one output and vice-versa) are called one-to-one.

- Write this equation in standard form: $ax + by = c$.

  Answer: $2x - y = -100$

Have students return to the situation that the equation represents and use the sentence frames below to apply the language of functions to the example.

- In the example, y represents . . .
- In the example, x represents . . .
- In the example, it makes sense to say that 'y is a function of x' because . . .

Appendix B: Working with Inequalities

Use the information below to teach students about inequalities.

Tell students that solving inequalities in two variables is much like solving inequalities in one variable.

For example, to solve the one-variable inequality below, first isolate the variable. Then mark the number line to represent the point that splits the number line into two: to the right lie the values that are greater than –2, and to the left lie the values that are less than –2. Shade the region of the number line that satisfies the original inequality and determine whether –2 is in fact part of the solution:

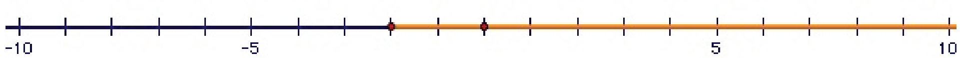
$$x + 5 \geq 3$$

$$\overbrace{x + 5}^{\text{this expression represents distance from -5}} \geq 3$$

Subtracting 5 from both sides of the inequality yields

$$x \geq -2$$

The solution to the inequality is the set of all numbers that are greater than or equal to –2.



For two-variable inequalities, graph the function on the coordinate plane and shade the appropriate region of the plane.

When working with a linear function, it is helpful to first change the equation into slope-intercept form. Then, determining which region to shade reduces to checking the relationship between *x*- and *y*-coordinates of points on the plane.

For example, use the inequality written in standard form:

$$2x + y \geq 5$$

Solving for *y* yields,

$$y \geq -2x + 5$$

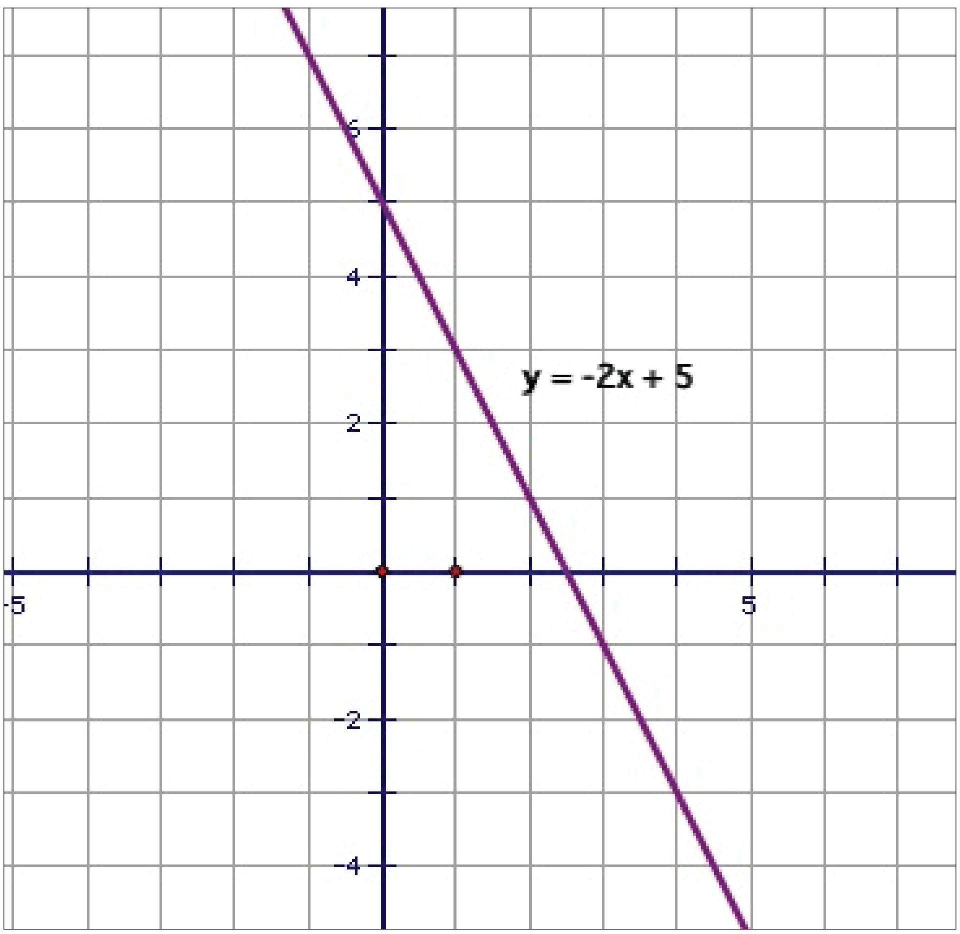
To represent this inequality on the coordinate plane,

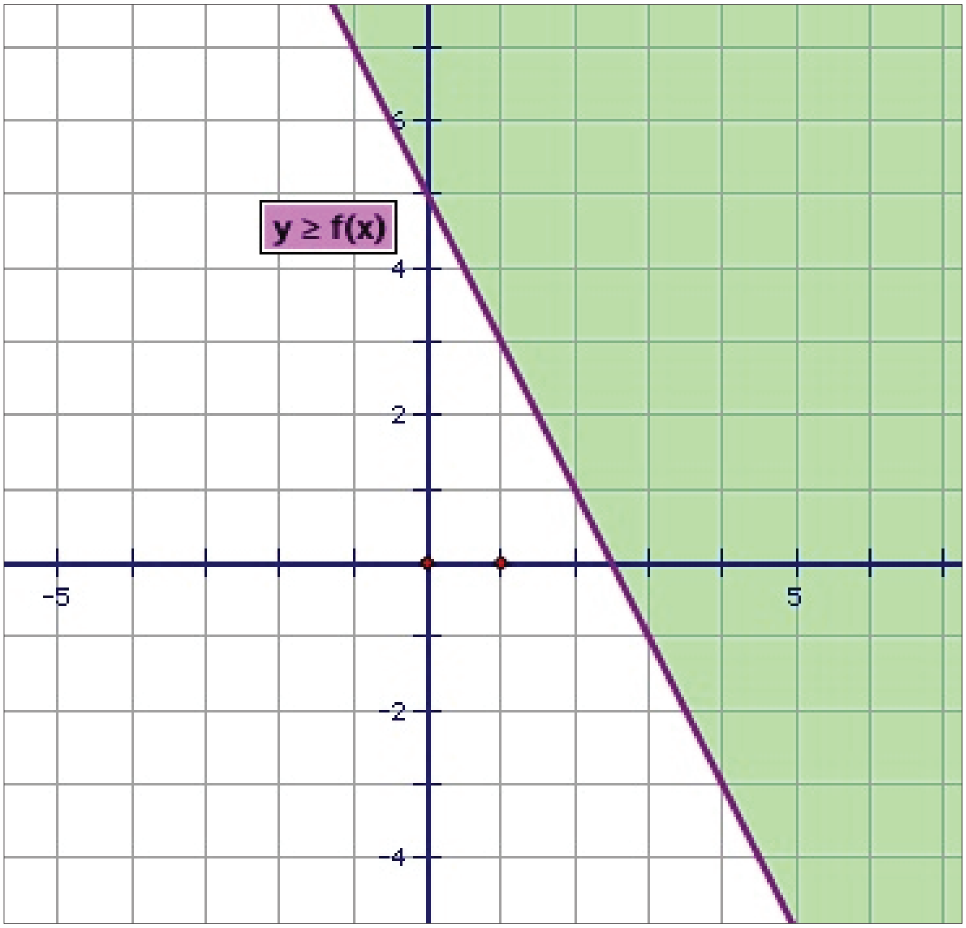
- first graph the linear equation
- then determine which part of the plane satisfies the original inequality and shade that part.

Use a test point to determine which part of the plane to shade. Pick the coordinates of a point and test to see whether the point satisfies the original inequality. If it does, then all points on the same part of the plane will satisfy the inequality. Shade that part of the plane to represent all possible solutions.

Be sure to discuss with students the cases when the line representing the function is shown as a dashed line (for inequalities with < and >) and when it is shown as a solid line (for inequalities with ≤ and ≥).

$y = -2x + 5$





Remind students that when working with inequalities, multiplying or dividing by a negative number causes the sign of the inequality to change. Show students why this occurs by working with a simple inequality, such as $-2 \leq 4$.

Appendix C: Solving Systems of Linear Equations

Use the information below to teach students how to solve systems of linear equations.

Introduce students to the process of solving a system of two equations in two unknowns by using the *elimination* method, also called the *addition-multiplication* method.

Teacher's Notes: The Elimination Method

Students may be familiar with the elimination process for solving a system of two equations in two unknowns, and they may also know how to use substitution. Any of these methods can be applied to linear programming problems.

The purpose of this activity is to help students understand *how* the elimination process works by investigating graphs at various steps.

1. Start with a simple system of two linear equations.
Explain that you solve the system of two linear equations in order to find the coordinates of the corner points of the feasible region in a linear programming problem.

Distribute **Handout: Solving Systems of Linear Equations**. Use the handout as you guide students through the elimination method of solving a system of two equations.

Students can work individually or with a partner as you model the work on the handout.

Note: You might want to have students work on solving additional systems of linear equations in order to better prepare students before they return to the linear programming problems in the unit.

2. Consider types of solutions to a system of linear equations.
Use the prompts below to have students begin thinking about types of solutions to systems of linear equations.

- Solving a system of linear equations is equivalent to finding the point(s) where lines intersect. Sketch a graph that represents a situation where there is **no solution** to a system of linear equations.

  Possible answer:
  A graph where two or more lines are parallel.

- Sketch a graph that represents a situation with an **infinite number of solutions**.

  Possible answer:
  A graph where one line is the same as another line; this occurs when the equation of one line is a multiple of the other line.

Handout:
Solving Systems of Linear Equations

You are going to learn about the elimination method for solving systems of equations. You can apply this method to find the coordinates of the corner points in the feasible region for Problem A—Media Selection. Then you can go on to solve Problem A.

Use a separate sheet of paper for your calculations and one sheet of graph paper with coordinate axes.

Solve the following system of two equations with two unknown variables:

$$5x - 2y = 10 \qquad (1)$$

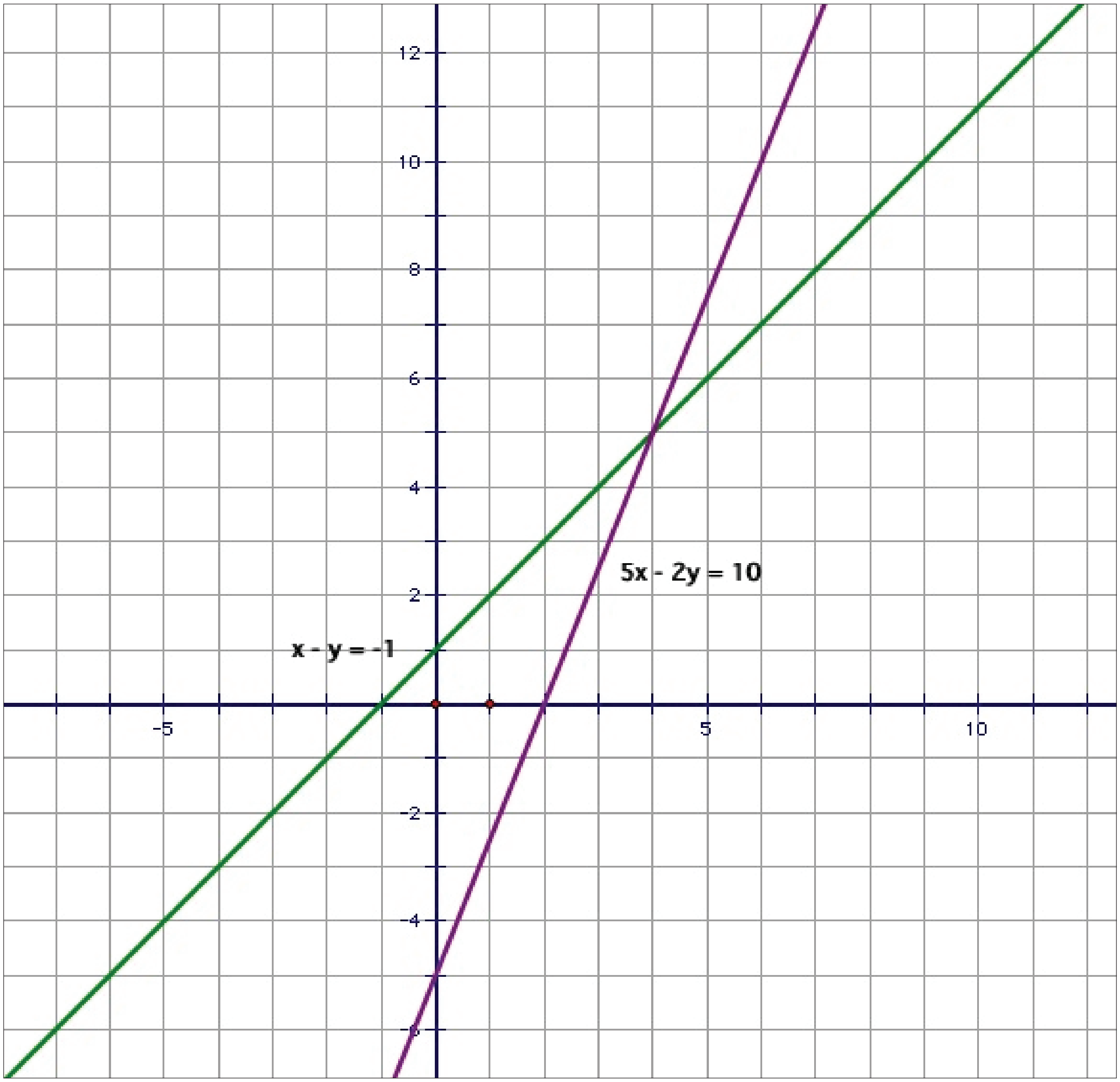
$$x - y = -1 \qquad (2)$$

1. What does it mean to solve a system of two equations? Recall that solving one equation with one unknown variable means finding the value(s) that makes the equation true.

   Answer: To solve a system of two equations means to find a point (or points) that satisfies both equations; graphically, the point(s) lies on both lines and is the point of intersection of the two lines, if a point of intersection exists.

2. Graph the two linear equations on the same coordinate plane. It may be helpful to use *x*- and *y*-intercepts when graphing each line.

   Answer: The graphs of the two lines on the same plane are shown on page 101.



12
10
8
6
4
2
-2
-4
-5
5
10
5x - 2y = 10
x - y = -1

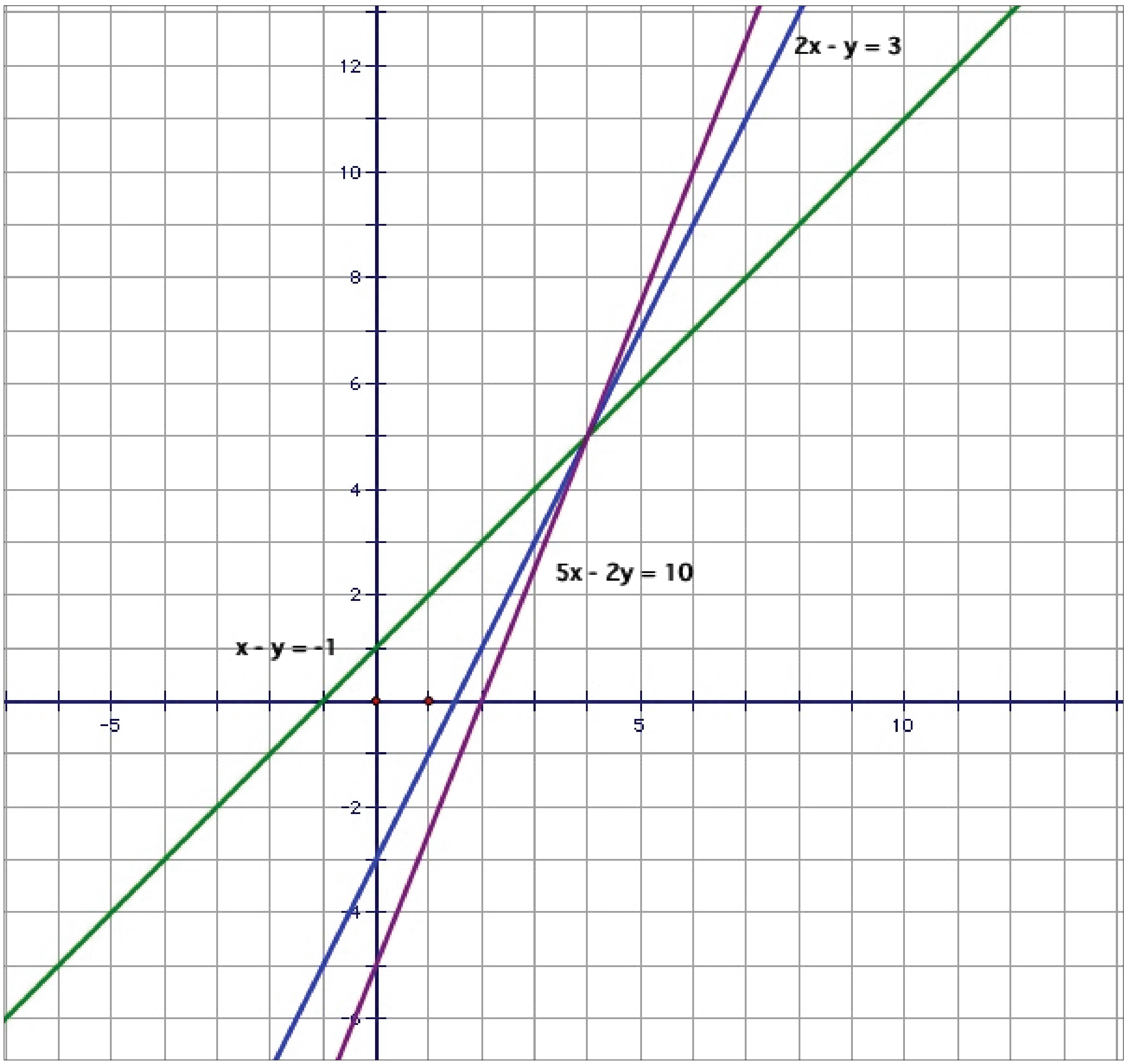
3. Line up the two equations and add them, term by term. Graph the resulting equation on the same coordinate plane. Be sure to label each line on your graph.

Answer:

$$\begin{array}{r} 5x - 2y = 10 \\ x - y = -1 \\ \hline 6x - 3y = 9 \end{array}$$

Divide each term in the new equation by 3: 2x – y = 3.

The graph of this line is shown below.



4. Multiply equation (2) by 3 and add your result to equation (1). Graph this equation on the same coordinate plane.

   Answer:

   3(x – y = –1)

   3x – 3y = –3

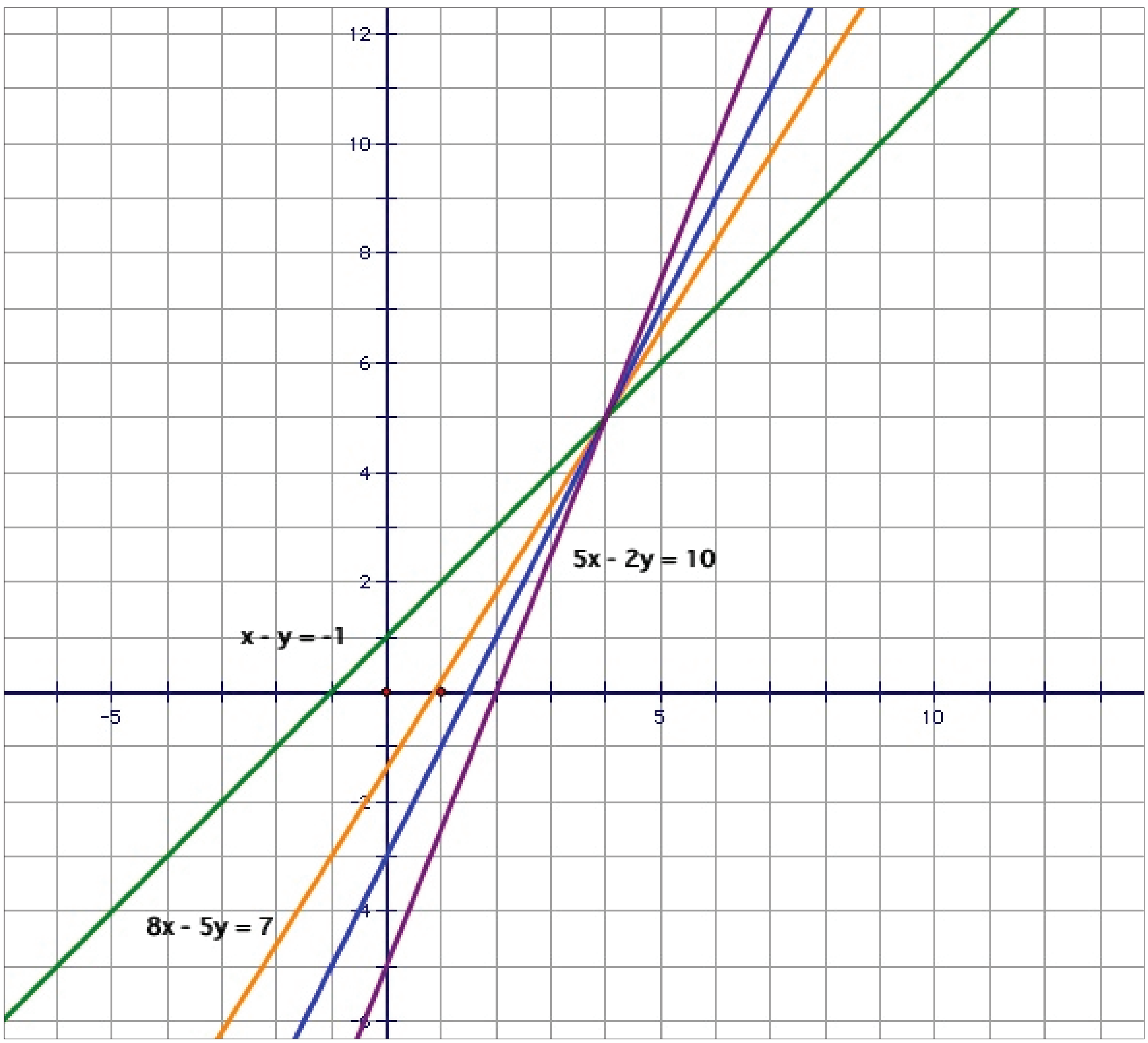
   Adding the result to equation (1):

   5x – 2y = 10

   3x – 3y = –3

   8x – 5y = 7

   The graph of this line is shown below.



5. Multiply (2) by –2 and add the result to equation (1). Graph this equation on the same coordinate plane.

Answer:

$-2(x - y = -1)$

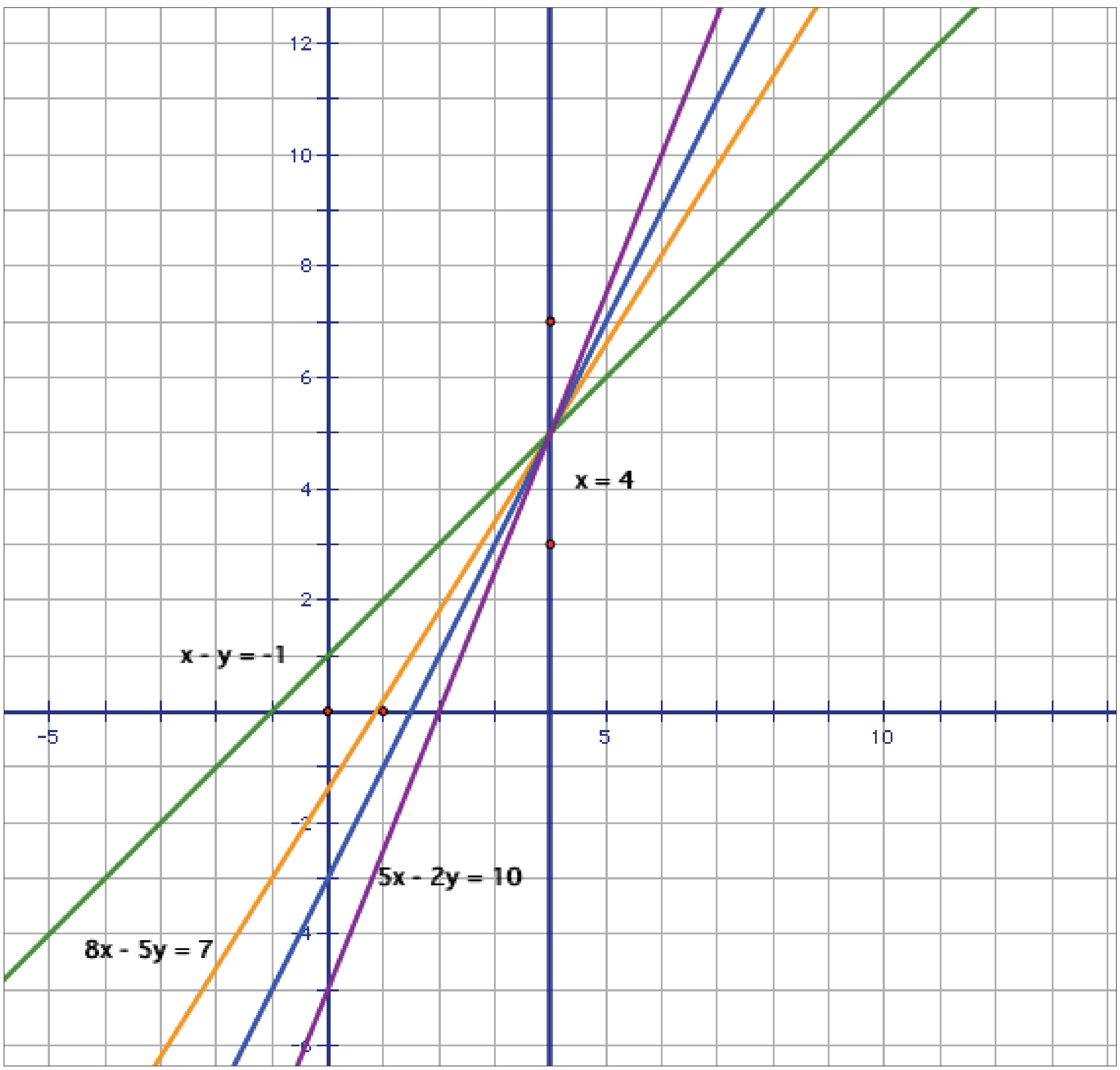
$-2x + 2y = 2$

Adding the result to equation (1):

$$\begin{array}{r} 5x - 2y = 10 \\ -2x + 2y = 2 \\ \hline 3x \quad\quad = 12 \end{array}$$

$x = 4$

The graph of this line is shown below.



6. As a last step in solving this system of equations, multiply equation (2) by –5 and add the result to equation (1). Why is this the last step?

   Graph the new equation and record your observations. Review your work in Steps 1 through 6 and prepare to make generalizations about the elimination process in Step 7 below.

   Answer:

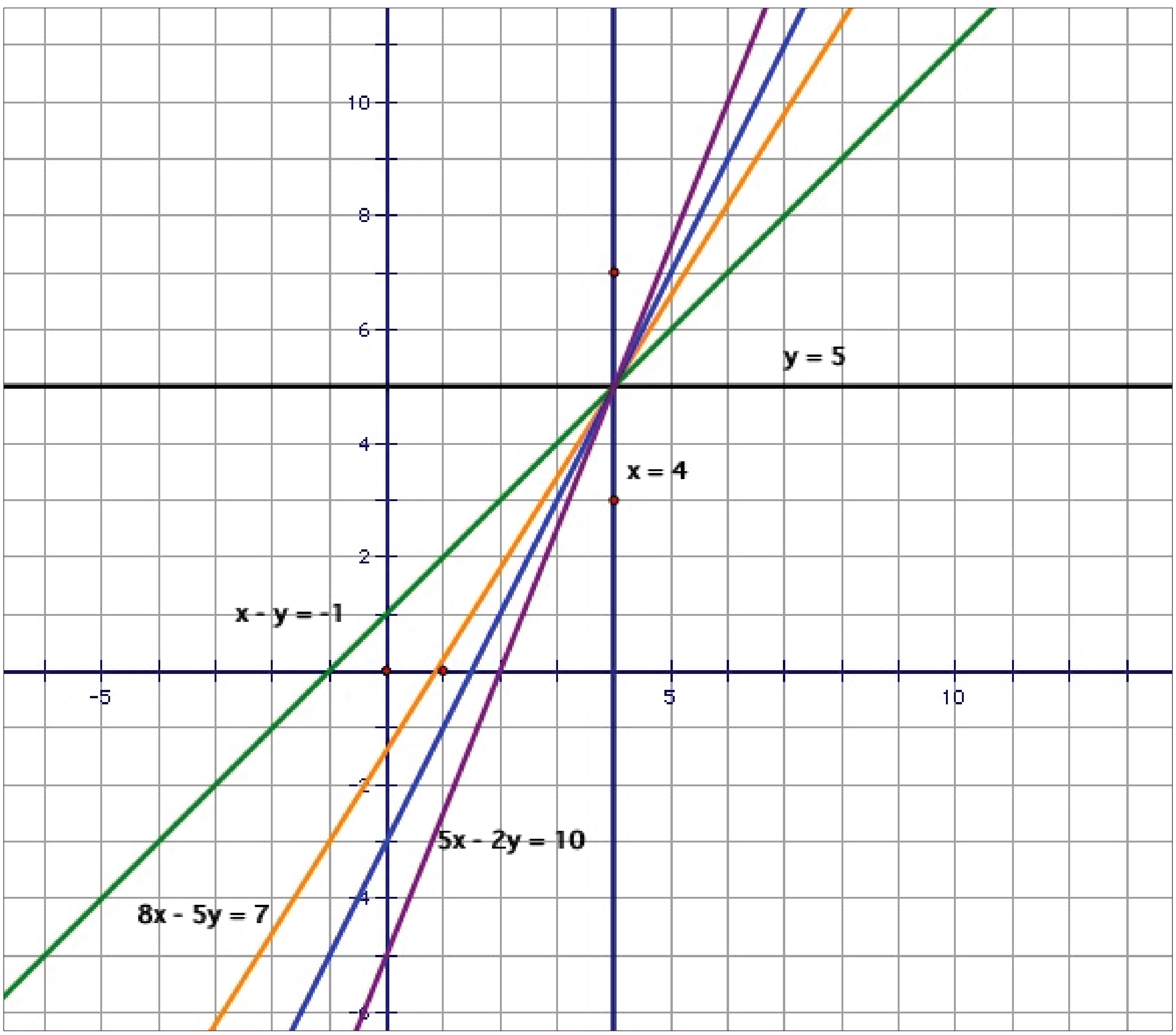
   –5(x – y = –1)
   –5x + 5y = 5

   Adding the result to equation (1):

   $$\begin{array}{r} 5x - 2y = 10 \\ -5x + 5y = 5 \\ \hline 3y = 15 \end{array}$$

   y = 5

   The graph of this line is shown below.



7. Each line that you graphed is a *linear combination* of the two lines with which you started. A *linear combination* is a sum of multiples of linear equations. You obtained each new line by multiplying one of the original lines and adding it to the other.

   (a) What is common among all the linear combinations of the two lines?

   (b) Do you think this is true for all cases?

   (c) What is the solution of the problem on this handout? Which of the equations you generated is satisfied by the solution?

   (d) When solving a system of linear equations, some combinations are more useful than others. Which linear combinations were useful in obtaining the point of intersection?

   (e) Write a procedure that explains the process and rationale of the elimination method. If necessary, begin with an example and explain the method as you proceed through its various steps.

Appendix D: Extension for Problem A—Media Selection

Use the information below to give students an opportunity to do additional work with Problem A—Media Selection.

You can give students alternative formulations of Problem A—Media Selection. These include:

- using different types of media vehicles (other than radio spot ads and newspaper ads)
- changing the objective function
- omitting one of the constraints in the problem

Teacher's Notes: Alternative Objective Functions

You can provide students with information about exposure quality ratings to have them think about alternative objective functions for Problem A.

For example, a different objective function might be to maximize the *value* of the promotion plan rather than the reach of the plan. The value of the promotion plan could be determined by using the exposure quality index.

The activity below provides information on using an exposure quality index to measure the effectiveness of advertising media.

Making Sense of the Exposure Quality Rating System

After consulting a marketing specialist, the promotion committee decided to make use of an *exposure quality rating* system. The consultant developed an index that measures *exposure per advertisement*, on a scale from 0 to 100, with greater numbers corresponding to higher exposure value of the ads. The exposure quality index takes into account audience demographics, including age, income, and education of the audience reached, as well as the image presented and the quality of the advertisement.

The value of the index for local newspaper advertising is 50.

The value of the index for spot radio advertising is 80.

Help the Wide Angle Youth Media promotion committee determine the combinations of newspaper ads and sets of radio spots that together will *average* an exposure quality value of 70.

- Assign variables: What are the unknown quantities? Use letters to represent these variables.

  Possible answer:

  Let r = *number of sets of 3 radio spot ads*
  Let n = *number of newspaper ads*

- Look for relevant information: What do you need to know about exposure quality values?

  Possible answer:

  The exposure quality value for radio advertising is 80; each set of 3 radio spot ads carries a weight of 80 using this index.
  The exposure quality value for newspaper advertising is 50; each newspaper ad carries a weight of 50 using this index.

- Write an equation that represents the combinations of radio spot ads and newspaper ads that together achieve an exposure quality value of 70.

  Possible answer:

  *An expression that represents the quality exposure value of r sets of radio spots is 80*r.

  *An expression that represents the quality exposure value of n newspaper ads is 50*n.

  The total number of sets of radio spots and news ads is (r + n)*. Thus, to average an exposure quality value of 70, write the following equation:*

  $$\frac{80r + 50n}{r + n} = 70$$

  Since r + n *≠ 0, we can multiply both sides of the equation by the expression* (r + n) *to obtain:*

  $80r + 50n = 70(r + n)$
  $80r + 50n = 70r + 70n$

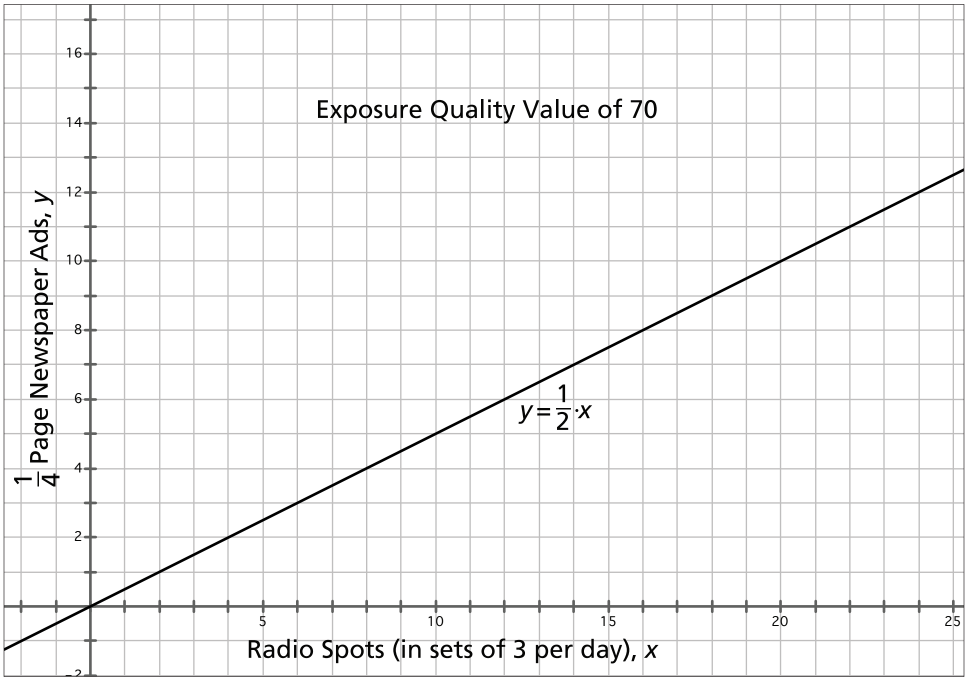
  Combining like terms yields

  $10r - 20n = 0$

- Graph the equation using the intercepts.

  Answer:

  The graph of the linear equation is shown below.



- Interpret the meaning of the graph.

  Possible answer:

  *The points that lie on the linear equation 10*r *– 20*n *= 0 represent all of the possible combinations of news ads and radio spot ads that average an exposure quality value of 70. This equation can also be written:*

  $$n = \frac{1}{2} r.$$

  Thus, as long as the number of newspaper ads is one half of the number of sets of radio spot ads, an exposure quality value of 70 is achieved.

Materials Needed

Throughout Unit

- Graph paper and rulers

Part 1: Problem Design

Supplies and Equipment

- Framing questions on chart paper (see *Advance Preparation*)

Handouts

- **Handout 1: Unit Overview**
- **Handout 2: Problem A—Media Selection**
- **Handout 3: Objective Match-Up**
- **Handout 4: Problem A—Media Selection: More Information**
- **Handout 5: Linear Functions and Their Representations**
- **Handout 6: Assembling Your Portfolio**
- **Assessment Checklist: Unit Portfolio**

Media Resources

- Blank graphic organizer (one per group)
- Completed graphic organizer containing information from Problem A on Handout 4 (see *Advance Preparation*)
- Example linear programming problems (see *Media & Resources*)

Advance Preparation

- Before Activity 1B.2, create a completed graphic organizer that organizes the information from Problem A on Handout 4. See *Media & Resources* for links to blank graphic organizers, and see page 24 for a sample completed graphic organizer.
- Before Activity 1C.1, write the framing questions for the unit on chart paper:
  - What do I need to know about a situation in order to formulate a useful problem?
  - How can a mathematical model provide insights into a real-world problem?
  - In particular, how can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost)?
  - How can the linear programming process help me promote and set up an event such as a visual arts and media event?
- Before Activity 1C.2, gather examples of linear programming problems for students. (See *Media & Resources* for examples.)

Part 2: Working with Constraints

Supplies and Equipment

- Graph paper or transparencies with grids (see *Advance Preparation*)
- Colored pencils or transparency markers (see Advance Preparation)
- Supplies to create consolidated graphs:
  - Colored dry-erase markers and large sheets of laminated graph paper (one per pair) (if students created their Handout 7 graphs on sheets of graph paper)
  - Transparency markers and transparencies with grids (one per pair) (if students created their Handout 7 graphs on transparencies)

Handouts

- **Handout 7: Problem B—Profit Maximization**
- **Handout 8: Problem B—Profit Maximization: The Feasible Region**
- **Handout 9: Problem A—Media Selection: Mathematical Model**

Media Resources

- Example linear programming problems (one problem per pair—see *Advance Preparation*)

Items Students Need to Bring

- Students' copies of **Handout 2: Problem A—Media Selection** from Part 1
- Students' copies of **Handout 4: Problem A—Media Selection: More Information** from Part 1
- Students' work on Handouts 2 and 4 (from Activities 1A.2 and 1B.1)
- Students' completed copies of **Handout 5: Linear Functions and Their Representations** from Part 1
- Students' graphs (created on paper or on transparencies) from **Handout 7: Problem B—Profit Maximization**

Advance Preparation

- Before Activity 2A.1, decide whether you will have student pairs use sheets of graph paper or gridded transparencies to graph the constraints in the linear programming problem. You may want to have students graph each constraint on a separate transparency in preparation for identifying the feasible region in Activity 2B.
- Make enough transparencies so that you can distribute at least four to each pair. You can create the transparencies by printing a graph-paper grid template onto printer-friendly transparencies. Have transparency markers available for students to write on the transparencies. If students are instead using sheets of graph paper to create their graphs, have colored pencils available.
- Before Activity 2A.3, choose several different linear programming problems to use as examples. Make enough copies so that you can

distribute one problem to each pair of students. (It's fine if some pairs work with the same problem, as long as each pair has its own copy to work with.) (See *Media & Resources* for example problems.)

Part 3: Getting to a Solution

Supplies and Equipment

- Transparencies with grids
- Transparency markers
- Optional: Laminated graph paper or chart paper with grids
- Optional: Cardboard strips
- Optional: Computers (for writing unit reflection)

Handouts

- **Handout 10: Problem B—Profit Maximization: Optimal Solution**

Items Students Need to Bring

- Students' copies of Handouts 1–10 from Parts 1 and 2
- Student's work and graphs related to the handouts from Parts 1 and 2

Advance Preparation

- Before Activity 3A.2, decide whether to review with your students how to solve systems of linear equations. Students will need to know how to solve them in order to find the solution for Problem A—Media Selection. You can use **Appendix C: Solving Systems of Linear Equations** for a review.

Media & Resources

These recommended Web sites have been checked for availability and for advertising and other inappropriate content. However, because Web site policies and content change frequently, we suggest that you preview the sites shortly before using them.

Media & Resources are also available at http://dma.edc.org and at http://dmamediaandresources.pbworks.com, a Wiki that allows users to add and edit content.

Mathematics Resources

Linear Inequalities and Linear Programming

Saul Gass (1970). An Illustrated Guide to Linear Programming. Published by McGraw-Hill, New York.

The Interactive Mathematics Program Year 2 text includes a complete unit on linear programming entitled *Cookies*, pp. 301–375. One particularly useful section within this unit occurs on days 22–27 of the suggested schedule. Here students are led through the process of creating their own linear programming problems.

The Interactive Mathematics Program is an NSF-funded curriculum. For ordering information, see Key Curriculum Press:

www.keypress.com/x5436.xml

Solving Systems of Equations

At the *Insights for Algebra 1* Web site, the lesson entitled "Left Hand, Right Hand" offers an interesting activity that asks students to collect data on the time it takes them to write letters first using their right hand and later their left hand. Students then enter their data into TI graphing calculators and find lines of best fit for each set of data. In groups, students discuss their results and determine the meaning of intersection points between their lines. They then consider the three scenarios for solving systems of equations with respect to the activity—when there is one solution and the two lines intersect at a point, when the two lines are parallel, and when two lines coincide. (In this activity, if the two lines coincide, then the student is ambidextrous!) This lesson plan can be found at:

www.learner.org/channel/workshops/algebra/workshop3/lessonplan1b.html

ExploreLearning has created a Gizmo™ with an accompanying activity that models a cat and mouse race. The speed that each travels (slope) as well each animal's starting point (*y*-intercept) are specified and the user simulates the race to determine whether the cat catches the mouse (the point of intersection). Students can change parameters (such as speed and starting point) to gain a clear understanding of the situation and the mathematical model. ExploreLearning Gizmos™ can be accessed at www.explorelearning.com. This particular activity is found at www.explorelearning.com/index.cfm?method=cResource.dspExpGuide&ResourceID=108.

"Illuminating Elimination":
www.pbs.org/mathline

A linear programming applet that is useful for understanding the Corner Point Theorem can be found at: www.exploremath.com/activities/Activity_page.cfm?ActivityID=31.

COMAP/HistoMAP, Module 20: *Optimality Pays: An Introduction to Linear Programming*, by Jeganathan Sriskandarajah. Published by COMAP, Inc., 1992.

Media-related Resources

An Introduction to Management Science: Quantitative Approaches to Decision Making, by D. Anderson, D. Sweeney, and T. Williams. Published by Thomson Learning, South Western, 2005. This is a college-level text for the management sciences that dedicates four chapters to linear programming, providing specific examples in media selection and scheduling.

A Power-point presentation introducing the media-planning field can be found at
www.londremarketing.com/documents/Media12062005.ppt

Part 1: Problem Design

Activity 1B.1: Revisiting Problem A—Media Selection

Graphic Organizers

The following Web sites include blank graphic organizers:

edHelper.com

http://edhelper.com/teachers/graphic_organizers.htm

Education Place

www.eduplace.com/graphicorganizer/

North Central Regional Education Laboratory

www.ncrel.org/sdrs/areas/issues/students/learning/lr1grorg.htm

TeacherVision

www.teachervision.fen.com/graphic-organizers/printable/6293.html

teAchnology

www.teach-nology.com/worksheets/graphic/

Thinkport

www.thinkport.org/technology/template.tp

Activity 1C.2: Partner Work

Examples of Linear Programming Problems

For a series of linear programming problems geared towards high school algebra students, you can order *Does This Line Ever Move? Everyday Applications of Operations Research*, by Kenneth Chelst and Thomas Edwards, available at www.keypress.com/x5991.xml. The text provides case studies of real problems from business and industry that use linear programming methods for optimization. These case studies are also available on the High School Operations Research Web site:

www.hsor.org/case_studies.cfm

Part 2: Working with Constraints

Activity 2A.3: Partner Work—Problem Formulation

See Activity 1C.2 above for a source for examples of linear programming problems.

Additional Resources for Teachers

Part 2: Working with Constraints

Activity 2A: Organizing the Constraints

These sites provide information about technology tools that can be used to work with linear inequalities.

Resources for use with The Geometer's Sketchpad®:

www.keypress.com/x6481.xml

Resource for graphing inequalities in two variables using the Texas Instruments graphing calculators:

www.keypress.com/documents/da2/CalculatorNotes/TI83-84Plus/DA_TI83-84Plus_05.pdf

Free guidebook on graphing inequalities with the TI-83 Plus:

http://education.ti.com/downloads/guidebooks/apps/83inequality_graphing/ineq-eng.pdf

Standards

This unit was developed to meet the following standards.

California Academic Content Standards for Mathematics, Grades 9–12

- Students simplify expressions before solving linear equations and inequalities in one variable [*Algebra 1, 4.0*]
- Students solve multi-step problems involving linear equations and inequalities in one variable [*Algebra 1, 5.0*]
- Students graph linear equations and linear inequalities [*Algebra 1, 6.0*]
- Students solve a system of linear equations or linear inequalities algebraically and interpret the answer graphically [*Algebra 1, 9.0*]
- Students understand the concepts of a relation and a function [*Algebra 1, 16.0*]
- Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices [*Algebra 2, 2.0*]

CTE AME Industry Sector Foundation Standards

4.0 Technology

Students know how to use contemporary and emerging technological resources in diverse and changing personal, community, and workplace environments:

> **4.2** Understand the use of technological resources to gain access to, manipulate, and produce information, products, and services.
> **4.7** Understand how technology can reinforce, enhance, or alter products and performances.

11.0 Demonstration and Application

Students demonstrate and apply the concepts contained in the foundation and pathway standards.

NCTM Standards

- Students understand the meaning of equivalent forms of expressions, equations, inequalities, and relations [*Algebra*]
- Students write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases [*Algebra*]
- Students use symbolic algebra to represent and explain mathematical relationships [*Algebra*]
- Students build new mathematical knowledge through problem solving [*Problem Solving*]
- Students solve problems that arise in mathematics and in other contexts [*Problem Solving*]
- Students monitor and reflect on the process of problem solving [*Problem Solving*]
- Students communicate their mathematical thinking coherently and clearly to peers, teachers, and others [*Communication*]
- Students use the language of mathematics to express mathematical ideas precisely [*Communication*]
- Students recognize and use connections among mathematical ideas [*Connections*]
- Students create and use representations to organize, record, and communicate mathematical ideas [*Representation*]
- Students select, apply, and translate among mathematical representations to solve problems [*Representation*]

Bibliography

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