INTEGRATED MATHEMATICS UNIT

TEACHER GUIDE

FUNCTIONS AND SOUND

MATH

DIGITAL MEDIA ARTS

dma.edc.org

EDC
Education Development Center, Inc.

the James Irvine foundation

ConnectEd
The California Center for College and Career

Education Development Center, Inc.

Carissa Baquiran, Kristen Bjork, Lisa Breit, Jen Clarke, Jennifer Davis-Kay, Jesse Dill, Maria D’Souza, Eliza Fabillar, Roser Giné, Vivian Guilfoy, Ilene Kantrov, Eric Karnowski, Rebecca Lewis, Emily McLeod, Kathleen McQuade, Cynthia Orrell, Elena Palanzi, Susan Richmond, Anne Shure, Fawn Thompson, Jason Tranchida, Zachary Yocum

Contact
Education Development Center, Inc.
55 Chapel Street, Newton, MA 02458-1060, USA
Phone: 617.969.7100 · Fax: 617.969.5979 · TTY: 617.964.5448
www.edc.org

ISBN
978-0-89292-613-8

Web Site
dma.edc.org

Development Partners

The James Irvine Foundation

Anne Stanton, Rogéair Purnell, Kathryn Furano, Matt Kelemen

ConnectEd: The California Center for College and Career

Gary Hoachlander, Paula Hudis, Pier Sun Ho, Khahn Bui, Dave Yanofsky

Advisors

Industry and Community Advisors

Deborah Brooks
The ACME Network

Milton Chen, PhD
The George Lucas Educational Foundation

Marilyn Friedman
DreamWorks Animation LLC

Pete Galindo
Independent Video Consultant and Educator

Kate Johnson
EZTV

Melissa Malinowsky
Independent Photo Editor

Erik Mason
Imaginary Forces

Dave Master
The ACME Network

Kathleen Milnes
The Entertainment Economy Institute

Dan Norton
Filament Games

Scot Osterweil
The Education Arcade

John Perry
The ACME Network

Chris Runde
Bay Area Video Coalition (BAVC)

Jessica Sack
Yale University Art Gallery

John Tarnoff
DreamWorks Animation LLC

Moriah Ulinskas
Bay Area Video Coalition (BAVC)

Eric Zimmerman
Gamelab

Secondary Educators and Pilot Teachers

*We are particularly grateful for the suggestions and guidance of the teachers who pilot-tested the curriculum.

Rosa Anaya*
John Muir High School, Pasadena, CA

Joel Buringrud*
Harmony Magnet Academy, Strathmore, CA

Richard Burrows
Arts Education Branch,
Los Angeles Unified School District

Pam Carter
Santa Susana High School, Simi Valley, CA

Deborah Claesgans
Arts Education Branch,
Los Angeles Unified School District

Cathee Cohen
Grover Cleveland High School,
Los Angeles, CA

Heidi Cregge*
Oakland School for the Arts, Oakland, CA

Barrington Edwards
Boston Arts Academy, Boston, MA

Virginia Eves
Office of College, Career & Technical
Education, San Diego Unified School District

Soma Mei-Sheng Frazier
Oakland School for the Arts, Oakland, CA

Shivohn Garcia
Paul Cuffee School, Providence, RI

Lorena Guillen*
John Muir High School, Pasadena, CA

John Hammelmann*
Harmony Magnet Academy, Strathmore, CA

Scott Hebeisen*
Digital Media Design HS, San Diego, CA

Brianna Larkin*
Oakland School for the Arts, Oakland, CA

Shawn Loescher
Office of College, Career & Technical
Education, San Diego Unified School District

Gail Marshall*
Van Nuys High School, Los Angeles, CA

Matt Maurin*
Edison High School, Stockton, CA

Jack Mitchell
California Department of Education

Frank Poje
History-Social Science Educator

Christina Ricard
Murdock Middle/High School,
Winchendon, MA

Nicholas Rogers
Career Development Unit, DACE,
Los Angeles Unified School District

Mark Rosseau*
Richmond High School, Richmond, CA

Shawn Sullivan
Sheldon High School, Elk Grove, CA

David Wilson*
Cesar Chavez High School, Stockton, CA

Jose Velazquez*
Harmony Magnet High School, Strathmore, CA

Post-Secondary Educators

Kristine Alexander
The California Arts Project,
California State University

John Avakian
Community College Multi-media
and Entertainment Initiative
College of San Mateo, CA

Brandi Catanese
University of California, Berkeley

Elizabeth Daley
School of Cinematic Arts,
University of Southern California

Amy Gantman
Otis College of Art and Design, CA

Evarist Giné
Professor of Mathematics,
University of Connecticut

Samuel Hoi
Otis College of Art and Design, CA

David Javelosa
Santa Monica Community College, CA

Jack Lew
Center for Emerging Media,
University of Central Florida

Sue Maberry
Otis College of Art and Design, CA

Tara McPherson
University of Southern California

Carol Murota
University of California, Berkeley

Casey Reas
University of California, Los Angeles

Carl Rosendahl
Carnegie Mellon University-
Silicon University Campus

Guy Smith
Santa Barbara City College, CA

Matt Williams
Institute for Multimedia Literacy,
University of Southern California

Holly Willis
Institute for Multimedia Literacy,
University of Southern California

Ellen Winner
Project Zero,
Harvard Graduate School of Education, MA

Contents

Unit Overview

In this unit, students discover how sound waves can be modeled mathematically. After a brief overview of the mechanics of sound, students learn that the graph of a sine function is a wave, much like a sound wave, so sound waves can be modeled using sine functions. Students investigate and compare quadratic, exponential, logarithmic, and sine functions and the equations and graphs that represent them. By experimenting with these equations, students understand how variables and constants affect the shapes of their graphs, and then apply what they have learned to generate tones that differ in pitch and volume.

Unit Length

7 50-minute sessions

Unit Project Description

For the unit project, students use the sine function as a model for sound waves. They work in pairs to predict how the parameters in different equations will produce a different wave and therefore a different sound, focusing on both pitch and volume. Using sound editing software, they apply the parameters from the equations to generate the sounds and then check their predictions. (If time allows, students may also use the frequencies for specific notes to write or recreate simple tunes.)

Assessment

Unit activities can serve as formative assessment tools. Use students' work to gather information about their progress and to identify concepts or skills to reinforce within your instructional practice. The following activities are particularly useful for formative assessment:

- **Handout 3: Finding the Wave Function** (Activity 1B)
- **Handout 4: The Graph of Cosine** (Activity 1B)

The project-based nature of the unit allows students to demonstrate their learning through authentic and relevant applications. This unit's summative assessment includes the following:

- **Handout 7: Sound Equations** (Part 3)

The Assessment Checklist provides criteria for assessment and a suggested weight for each. If you wish to use a rubric, work with teachers in your grade level or subject area to develop a tool that is consistent with your school's assessment system.

Framing Questions

- How can we mathematically represent an invisible phenomenon, such as sound?
- How are changes in a mathematical equation reflected in its graph?
- How can the volume and pitch of a sound be altered through changes in the function that represents that sound?

Understandings

- Changing the constants of a mathematical function moves and stretches its graph.
- Sound waves can be represented mathematically with trigonometric functions.
- Transformations of sine functions are directly related to changes in the pitch and volume of a sound.

Where the Unit Fits In

Functions and Sound is designed to be taught during Algebra 2, Trigonometry, or Pre-Calculus when students are studying trigonometric functions and their graphs. Students use mathematical expressions and graphs of the sine and cosine functions to model sound, including changes in pitch and volume. Understanding how sine functions and graphs model sound helps students connect mathematics to everyday life, and gives them a deeper understanding of audio recorders and sound editing software.

Integration with Foundations Courses

This unit integrates Algebra 2, Trigonometry, or Pre-Calculus content and career and technical education (CTE) knowledge and skills. It can be taught at the same time as or after the related unit in *Foundations in Media and Digital Design*.

***Foundations in Media and Digital Design: Audio & Video, Unit 1:* Using Sound to Tell Stories**

Students create an audio story, recording interviews and ambient sound, selecting music, and editing the clips to create a short piece. Discuss with the CTE teacher the visual displays of sound that students will see in their audio recorders and sound editing software. Share the mathematical terms that students will use—*sine function*, *amplitude*, *period*, *crest*, and *trough*—so the CTE teacher can reinforce this vocabulary and help students see how these science and math concepts relate to real-world situations.

Multi-Disciplinary Teams

This unit relates directly to the physics unit *Acoustics: The Science of Sound*, and students will benefit from their science and mathematics teachers teaching the units in a coordinated way. The history and English language arts (ELA) units are less directly related, although mathematics, physics, history, ELA, and CTE teachers may coordinate their teaching and have students work toward a pathway-wide multi-disciplinary project. Students can use their historical research and the techniques of memoir in their audio story scripts; they will apply their scientific and mathematical understanding of sound and sine functions as they record and edit sounds for their audio stories.

***Acoustics: The Science of Sound* (Physics).** *Functions and Sound* relates directly to this physics unit, in which students learn about the properties and behaviors of sound waves and mechanical waves in general. They create sound effects with sound editing software and write explanations of how wave properties, such as amplitude and frequency, were manipulated to produce the sound effect. They also write an illustrated article explaining a topic in audio production.

***Podcasting the Past* (U.S. History).** Students research places in their own communities and use their research to create the script for a podcast.

***Everyone Has a Story* (ELA).** Students learn to write about themselves by analyzing audio stories and excerpts from published memoirs and then developing their own short memoirs about a significant or dramatic event from their lives.

Student Prerequisites

Prior to beginning the unit, students should be able to do the following:

- Recognize and calculate trigonometric function (sine and cosine) values for common angle measures (30°, 45°, and 60°)
- Generate input-output tables for quadratic, cubic, and exponential functions

Table of Activities

Part 1: Making Waves (3 sessions)

Students are introduced to the physics of sound and to unit circle trigonometry, connecting these concepts through the idea of waves.

Activity 1A: Exploring Sound (1 session)

Students explore the nature of sound as it travels through air. Through a computer simulation, students learn how the volume and pitch of a sound can be adjusted. This exploration sets the stage for understanding parallels between changes in the graph of a sound wave and the resulting sound effects.

Activity 1B: Generating Sine and Cosine Graphs (2 sessions)

Students learn that sine and cosine waves can be used to represent tones. Students connect their prior knowledge of trigonometric ratios of acute angles within right triangles to the waveforms that result from graphing $y = \sin(x)$ and $y = \cos(x)$ on a coordinate system, using angle measure as the independent variable and the value of the trigonometric ratio in question as the dependent variable. Students create a cosine wave by hand, which allows them to focus on how the graph of the function is cyclical and appears as a wave and to better understand how the graph can model a sound wave.

Part 2: Making Changes (3 sessions)

Students work with transformations of functions to determine the effect of multipliers and added constants on the graphs of the functions. They start with algebraic functions and then extend their conclusions to the sine function.

Activity 2A: Transforming Functions (2 sessions)

To prepare for the kinds of changes they will apply to the equations in Activity 2B, students revisit the graphs and equations of known functions and explore how changes in the coefficients of a function's equation affect its graph. Types of changes explored include horizontal shifts (also called *phase shifts* with trigonometric functions), horizontal stretches (period adjustments), vertical stretches (changes in amplitude), and vertical shifts.

Activity 2B: Changing Sine (1 session)

Students apply the same transformations they used in Activity 2A to their graph of the sine function. Using graphing calculators, students observe how changing coefficients in the equation $y = \sin(x)$ affects the graph.

Part 3: Generating Sound (1 session)

Students use sound editing software to turn sine equations into sounds. They begin by determining the period, frequency, and amplitude of the waves represented by the equation. Next, they compare equations and their resulting sounds, making predictions about how the sounds and graphs will be different. Finally, they experience how changes in the waveforms affect the sounds they hear.

Advance Preparation

- Internet resources, provided in student handouts and as links in *Media & Resources*, are recommended throughout the unit for student or in-class use. These Web sites have been checked for their availability and for advertising and other inappropriate content. However, because Web sites' policies and content change frequently, we suggest that you preview the sites shortly before using them.
- Address any issues, such as firewalls, related to accessing Web sites or other Internet links at your school.
- Look at *Materials Needed* at the end of the unit and order or prepare any needed equipment or supplies.
- Part 3 requires students to use sound editing software during class. Reserve a computer lab and install sound editing software (such as the freely available program Audacity) on all computers. (See *Media & Resources* for links to this program.)

Part 1: Making Waves

Students are introduced to the physics of sound and to unit circle trigonometry, connecting these concepts through the idea of waves.

Length

3 50-minute sessions

Advance Preparation

- Before Activity 1A, gather materials you can use to demonstrate for students how sound is produced by vibrations from an object. Ideally, use objects that vibrate in a noticeable way—for example, pluck a guitar string or strike a tuning fork.
- Before Activity 1A, review **Handout 1: Exploring Sound** and the applets and simulations on the Web sites listed in the handout. See *Media & Resources* for more information about these Web sites.
- Before Activity 1B, preview the waveform applet, available online. See *Media & Resources* for a link to this applet.

RESOURCES

Activity 1A: Exploring Sound

Students explore the nature of sound as it travels through air. Through a computer simulation, students learn how the volume and pitch of a sound can be adjusted. This exploration sets the stage for understanding parallels between changes in the graph of a sound wave and the resulting sound effects.

Understandings

- Sound is a result of disturbances in the air caused by vibrations.
- The volume of a sound is determined by the amount of displacement caused by the vibrations.
- The pitch of a sound is determined by the frequency of the vibration (that is, how fast the air pressure changes from compression to rarefaction and back again).

Materials Needed

- Materials for the sound demonstration you will perform (see *Advance Preparation*)
- Computers with Internet access and speakers (one per pair of students, or one for the whole class with a projector)
- **Handout 1: Exploring Sound**
- **Handout 2: Unit Overview**

1. Conduct a demonstration of sound.

To set the stage for students' explorations, demonstrate sound by showing how objects vibrate in a noticeable way, for example, by plucking a stringed instrument or hitting a low-pitched tuning fork. Point out that these vibrations are how sounds are made. Students can even put their hands on their own throats and hum to feel the vibration.

Teacher's Notes: Science Integration

In the integrated unit *The Science of Sound*, students learn about the properties of sound waves. If students are taking or have taken that unit, you can relate the discussion here to what they have learned about the science of sound waves.

2. Have students use Web sites to explore the nature of sound.

Divide the class into pairs and give them **Handout 1: Exploring Sound**. Tell students that they will investigate the features of sound by interacting with sound demonstrations at two Web sites. Point out the two separate explorations on the handout and the instructions for using each Web site, and give students the URLs for the two sites (located in *Media & Resources*).

Have student pairs conduct each exploration, discussing and recording their observations and responding to the questions on the handout.

> **Note:** If time is limited, you can divide the class into two groups of pairs. Have the first group conduct Exploration 1 and the second group conduct Exploration 2 on **Handout 1: Exploring Sound**.

3. Ask students to share what they learned from their explorations.

Engage students in a whole-class discussion around their observations from the sound simulations. Ask students the following questions:

- What did you learn about how sound is created?

  Possible answers:
  - *Sound results from a disturbance in the air. Vibrations move air molecules and form a wave, which transmits the disturbance that we interpret as sound.*
  - *The disturbance in molecules causes* compressions*, regions of high air pressure, and* rarefactions*, regions of low air pressure. As the sound wave passes, the air molecules experience compressions followed by rarefactions; this causes the air molecules to move back and forth, though they remain near their original position once the wave has passed through them.*

- What seemed to affect the volume of a sound?

  Possible answers: *Larger initial displacement of the string caused a louder sound. Louder sounds caused the dots on the Illumination site to be darker; on the Sounds Amazing site, the sound wave graph was taller for a louder sound.*

  Note: Let students know that they're going to work more with sound waves during the unit, so the difference in height is going to come up again.

- What seemed to affect the *pitch* (whether a tone is high or low) of a sound?

  Possible answers: *Higher pitches had waves that were closer together.*

  Note: Students thinking about the Illuminations site may say that the "bubbles," or whatever term they use to describe each period, were smaller or narrower. You may need to help students connect their observation to the narrower waves from the Sounds Amazing site.

Have students record the class's answers in the Summary section of Handout 1. Let them know that they will need to refer to their notes in this section when they work on the unit project, as they connect the physics of sound to the transformations of function graphs.

4. Introduce students to the concepts they will learn about in this unit.
Tell students that they will now learn how to represent sound waves mathematically, using both graphs and equations. Give students **Handout 2: Unit Overview** and have them highlight *frequency* and *amplitude* in the vocabulary list.

Connect these vocabulary words to what they just saw on the Web sites:

- The *frequency* determines the pitch; in the visualizations, as the waves got narrower, the pitch got higher. You might point out that a full wave (from crest to crest) happens more frequently when the frequency is greater.
- The *amplitude* determines the volume of the sound; in the wave graph, sound with a greater amplitude appears as a taller wave.

Let students know that they will work with both of these ideas in the upcoming activities.

Handout 1:
Exploring Sound

Using computer simulations, you will explore the nature of sound as it travels through air. In particular, you'll consider what happens when the *pitch* of a sound changes and when the *volume* of a sound changes.

Exploration 1: Illuminations Sound Wave

Open the Sound Wave applet (your teacher will give you the URL).

There are two parameters that can be changed: the *initial string displacement* and the *tension* in the string. (Imagine that the string is a guitar string or one of a piano's strings.) The goal for this exploration is to determine how changes in these parameters affect the sound that is produced.

Step 1

Set the initial string displacement at a low value by moving the slider to the left. Click the "hear" button to hear the sound. Move the Initial String Displacement slider toward the right.

Write a description of how increasing the initial string displacement affects the sound. (Click the "hear" button again to stop the sound while you write your description.)

Answer: The initial string displacement affects the volume of the sound. The greater the displacement, the louder the sound.

Step 2

Move the Initial String Displacement slider to a middle value. Click the "start" button to start the string vibrating again. You will see dots that represent air molecules being moved by the string.

Describe the pattern you see for this single value of initial string displacement.

Answer: There are "bubbles" in which the dots are far apart, leaving a lot of open white space. As you get to the edge of a bubble, the dots get closer together, which makes the space look darker (and defines the edge of the bubble). The dots then get farther apart, creating the next bubble.

Step 3

Move the Initial String Displacement slider and notice how the pattern changes. You may want to compare a very low value (but not 0), a very high value, and a value in the middle.

Describe the change in the pattern.

Answer: The bubbles are lighter or less well defined when the value is low. They're much easier to see when the value is high.

Step 4

What do you think this change in pattern means for the air molecules? Write about how changing the Initial String Displacement changes the way the string vibrates.

Answer: High values make the string move more. The air molecules are pushed more, so they get pushed closer together at the edges of the bubbles.

Step 5

Move the Initial String Displacement slider to a high value. Press the "hear" button and move the String Tension slider. Listen to the changes in the sound produced.

Describe the effect of increasing the string tension on the sound that you hear.

Answer: The string tension affects the pitch of the sound heard. The greater the tension, the higher the pitch.

Step 6

Try this again, using the "start" button to see the representation of the air molecules. Move the String Tension slider and notice how the pattern changes.

Describe the effect of increasing the string tension on the pattern.

Answer: As the string tension increases, the bubbles get narrower. There are more bubbles in the same amount of time.

Exploration 2: Sounds Amazing Simulation

Go to the Sounds Amazing Web site (your teacher will give you the URL).

Click on "What is Sound?" on the left.

Read each slide and click the buttons for the simulations. When you're done with a slide, click the right arrow below it to go to the next page. (Note: Some slides have their own "next" arrow on them. Use those first.)

Stop when you're done with page 5, Loudness and Decibels.

Take notes on any vocabulary words and their meanings, and any other information you think is important.

Answer these questions:

- What do you notice about the relationship between pitch and the shape of the wave produced by a particular sound?

  Answer: The higher the pitch of the sound, the closer together the waves seem to appear.

- What do you notice about the relationship between volume and the shape of the wave produced by a particular sound?

  Answer: The louder the sound, the taller the resulting waves are.

Summary: The Nature of Sound

Record what you've learned about sound from your explorations and the class discussion. Take good notes—you'll need to refer to these ideas again later!

- What did you learn about how sound is created?
- What seems to affect the volume of a sound?
- What seems to affect the *pitch* (whether a tone is high or low) of a sound?

Handout 2:
Unit Overview

In this unit, you'll explore how sound works—from how it is created, to what affects its pitch and volume. You'll connect the physical properties of sound to the way it moves through the air, using mathematics to create graphs depicting sound waves. You'll also examine how changing the constants and coefficients in an equation can affect the graph of that equation, and ultimately how this can be used to create different tones.

Your work in this unit will revolve around the following questions:

- *How can we mathematically represent an invisible phenomenon, such as sound?*
- *How are changes in a mathematical equation reflected in its graph?*
- *How can the volume and pitch of a sound be altered through changes in the function that represents that sound?*

Unit Project

Once you've explored how sound can be modeled with waves, and how changing the parameters of an equation can affect its graph, you'll work with a partner to predict how the parameters in different equations will produce different waves—and therefore different sounds—focusing on both pitch and volume. You'll then check your predictions, using sound editing software. You may even choose to use this method to recreate a simple tune or to compose your own original musical work.

What You Will Do in This Unit

Explore how sound is created. Use computer applets to help you visualize how sounds are created and transmitted and how this process can be translated into a wave.

Give sound a mathematical context. Use your knowledge of right-angle trigonometry, and how it connects to circular functions and the unit circle, to create the graphs of sine and cosine functions.

Explore the effects of changing parameters on graphs. Change coefficients (multipliers) and add constants to several basic equations and compare the resulting graphs. Predict how changing the parameters of a function will affect its graph.

Manipulate waves. Apply what you discovered about parameter changes to sine functions, and connect this knowledge to how changes in parameters would affect the sounds modeled by sine functions.

Create sounds. Predict how sounds with different wave models will compare to each other, and test your predictions by using tools to produce sounds with given parameters.

Vocabulary Used in This Unit

Amplitude: (1) Half the difference between the maximum and minimum y-values of a sine or cosine wave. (2) The intensity of the air pressure caused by the vibrations that created the sound. This air pressure is measured in Newtons/meter^2 or decibels (dB), a logarithmic unit. Amplitude is related to the volume we perceive.

Compression: The crowding together of air molecules.

Cosine: A trigonometric function, using an angle measure as the independent variable. Cosine is defined using acute angles in right triangles as the ratio of the length of the adjacent leg to the length of the hypotenuse. It's represented in function notation as cos(*x*).

Crest: The point at which a wave reaches its highest value.

Cyclical: Recurring over and over at regular intervals, just as tracing a circle returns one to the same place over and over. Also referred to as *periodic* in mathematics.

Decompression (or rarefaction): Air molecules that are spread out.

Frequency of a sound wave: The speed at which the air pressure changes at a given point in space. The frequency of a wave is the number of cycles per unit time, or the number of cycles per second. Hertz (Hz) is the unit used to measure sound frequency and is defined as 1 cycle/second. Frequency is related to pitch in human perception. For example, a sound wave of 440 Hz sounds like the note A on a piano (just above middle C). Humans hear in a frequency range of about 20 Hz to 20,000 Hz.

Period of a sound wave: The time it takes to complete one cycle, measured in seconds/cycle. Frequency and period have an inverse relationship. If the *x*-axis represents time, the period is also the horizontal distance between consecutive crests (or troughs) in a sine or cosine wave.

Phase of a wave: The horizontal offset from the normal starting position of the basic wave $y = \sin x$ or $y = \cos x$ at $x = 0$.

Radian: A unit of measurement used for cyclical phenomena so that angles may be treated as distances. In a *unit circle* (a circle whose radius is 1 unit), a central angle that intercepts an arc whose length is 1 unit measures 1 radian.

Sine: A trigonometric function, using an angle measure as the independent variable. Sine is defined using acute angles in right triangles as the ratio of the length of the opposite leg to the length of the hypotenuse. It's represented in function notation as sin(*x*).

Sinusoidal: A graph or phenomenon that takes the shape of a sine wave, oscillating up and down in a regular, continuous manner.

Sound synthesis: This occurs when the changes that different sounds make in air pressure amplitude are combined and blended into one sound. When two sound waves travel through the same air space at the same time, their amplitudes at each moment in time are summed into a composite wave that contains the frequencies of each.

With an audio editing program, separate waves can be shown in different tracks. The editing software can then combine the waves into a single sound wave that has the frequency components of each wave.

Sound wave propagation: The passing along of the change in air pressure from air molecules to adjacent air molecules.

Trough: The point at which a wave reaches its lowest value.

Activity 1B: Generating Sine and Cosine Graphs

Students learn that sine and cosine waves can be used to represent tones. Students connect their prior knowledge of trigonometric ratios of acute angles within right triangles to the waveforms that result from graphing $y = \sin(x)$ and $y = \cos(x)$ on a coordinate system, using angle measure as the independent variable and the value of the trigonometric ratio in question as the dependent variable. Students create a cosine wave by hand, which allows them to focus on how the graph of the function is cyclical and appears as a wave and to better understand how the graph can model a sound wave.

Understandings

- Sound is a cyclical phenomenon, due to the nature of vibrations.
- Waves are a cyclical phenomenon.
- Since sine and cosine functions are also cyclical and have a wave graphical representation, these functions can be used to model sound.

Materials Needed

- Projection or printout of the graph in **Appendix A: Sound Wave Image**
- **Handout 3: Finding the Wave Function**
- Computers (one per pair of students, or one for the whole class with a projector) with Internet access
- Waveform applet (see *Advance Preparation*)
- **Handout 4: The Graph of Cosine**
- Graph paper
- Colored pencils
- Graphing calculators
- Rulers
- Protractors

1. Discuss the cyclical nature of sound.
Return to the observations students made during the first activity and emphasize the repetitious nature of the disturbances of air molecules. Tell students that the pattern of compression followed by rarefaction and back to compression of air molecules is *cyclical*, which means that it repeats at regular intervals as the sound source vibrates and the sound travels.

Show students a printout or projection of the visual representation of a simple sound wave in **Appendix A** and have them identify the following elements from the graph:

- The independent variable (time)
- The dependent variable (air pressure)
- Moments of high air pressure (crests)
- Moments of low air pressure (troughs)

Connect the crests and troughs with the "bubbles" that appeared in the Illuminations applet. The high-pressure moments are the compression moments, when the molecules are close together (the string presses them against each other), and this appeared in the applet as dense dots (where two bubbles met). The low-pressure (rarefaction) moments were when the molecules were farther apart, as the string moved away from the wave, and this appeared in the applet as an area with few dots (the center of each bubble).

Ask:

- What cycle do you notice in this graph?

  Possible answer: *The graph shows a cycle of crest and trough, then another crest and trough.*

- Do you notice any values that repeat in the graph?

  Possible answer: *The values 1, 0, and –1 repeat, as do all the* y*-values in the graph.*

Encourage students to share any other observations they have about the graph.

Remind students of two important ideas from the previous activity: *frequency* and *amplitude*. Ask them to suggest some ways that these two ideas appear in the graph. You don't need to verify or deny their suggestions, but collect as many ideas as you can. Tell them that these are two things they will figure out in this activity.

2. Have students explore the connection between circles and waves.
Give students **Handout 3: Finding the Wave Function** and have them work in pairs at computers.

Note: Alternately, you can display the waveform applet and have the class complete the activity as a group.

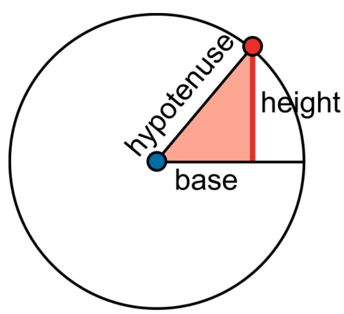
Show students how to open the waveform applet by navigating to the Web site where it is located. Have pairs complete Handout 3.

As students work, especially on Step 3, listen to their conversations. Note which pairs or groups recall the trigonometric functions, even if they don't remember enough detail to identify the sine function as the one that uses the side opposite the angle.

3. Discuss the connection between the sine function and the wave.
Have students share some of their answers to the questions on Handout 3. In particular, be sure to discuss their ideas for the functions that can be used to represent the wave. Let them explain their thinking, even if their answers are incorrect. If necessary, provide hints to help them recall trigonometric functions.

Review how the sine and cosine functions work, and verify with students that the sine function would provide the *y* value of the wave form, at least when the wave is above the *x*-axis (and therefore the *y* value is positive).

Sketch a circle on the board with a right triangle inside it, as students saw in the applet. Label the triangle's sides (base, height, hypotenuse):



Show students that the sine and cosine of acute angles are ratios of the sides of the triangle. Explain that $y = \sin(x)$ and $y = \cos(x)$ represent functions whose independent variable is the measure of an angle and whose dependent variable is the ratio of the sides of the right triangle. Thus, as the measure of an angle changes, so do the values of these two ratios. Emphasize that corresponding acute angles of similar triangles will have the same function values.

Point out that the graph of the sound wave from **Appendix A** is also a graph of the basic sine function, $y = \sin(x)$.

4. Introduce radian measure and negative trigonometric values.
Point out that as the dot traveled around the circle in the applet, particular triangles were created. On a unit circle, when the distance traveled along an arc is equal to the length of the radius, the angle of rotation has a measure of 1 radian. When the angle of rotation is measured in radians rather than degrees, the arc length of a unit circle is equal to the measure of that angle. The angle of rotation can be used as the input value for sine and cosine functions instead of the measure of an acute angle in a right triangle.

Tell students that when they work with waves and other cyclical patterns, *radian measure*, rather than degree measure, is commonly used for trigonometric functions. Explain that it is always assumed that the rotation starts on the right (the 3 o'clock position) and rotates counterclockwise, as was shown by the dot moving around the circle in the applet.

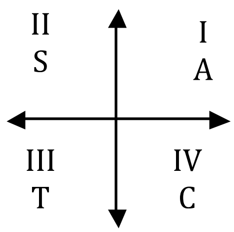
> **Note:** If you wish, you can tell students how to convert degree measure to angle measure: Point out that as the dot travels the full circle, it sweeps out a 360° angle. Have students supply the distance traveled (the circumference of a circle with radius 1) and use that to consider half-circles (180° and π radians) and quarter-circles.

Point out that when using triangles to work with trigonometric functions, the values are always positive (because they are ratios of lengths), and the angles are always acute angles.

Have students use either the applet or the graph of the sound wave from **Appendix A** to identify what happens to the sine value when the angles become obtuse, and then when the dot travels to past the half-circle (and the *y* values become negative). Point out the connection between the sine of the angle of rotation and the *y*-coordinate of the point on a unit circle where the terminal ray intersects it.

Introduce the quadrants and how the sign of the trigonometric value depends on which quadrant the value of the independent variable (in this case, time) puts the dot.

Teacher's Notes: A Mnemonic for Helping Students Learn



The following graphic and mnemonic device may be useful in helping your students remember the relationship between the quadrant in which a terminal ray lies and the sign of the angle's trigonometric values: ASTC (All Students Take Calculus).

In the first quadrant, all trigonometric functions have positive values.

In the second quadrant, sine and its reciprocal have positive values.

In the third quadrant, tangent and its reciprocal have positive values.

In the fourth quadrant, cosine and its reciprocal have positive values.

All trigonometric functions other than those named have negative values in each quadrant.

5. Have students generate the graph of cosine.
Give students **Handout 4: The Graph of Cosine**, along with graph paper, colored pencils, graphing calculators, rulers, and protractors. Have students complete Handout 4 individually or in groups. Tell students that this handout will give them a chance to work with cosine and see how it creates a wave graph.

> **Note:** Depending on students' recall of the trigonometric functions, you may first want to review special triangles (particularly 45°, –45°, and –90° triangles) and their cosine and sine values. You may also want to convert 45° to radians for or with the class.

Circulate to clarify the handout's instructions as students work.

6. Discuss Handout 4.
Assess students' understanding of Handout 4 by asking the following questions:

- What would happen to your graph of $y = \cos(x)$ if you continued cycling counterclockwise around the unit circle, using increasing angle measures?

  Possible answer: *The graph would repeat, creating a continuous wave.*

- What do you notice about the maximum and minimum values of the function?

  Possible answer: *They range between –1 and 1.*

- How long does it take for one cycle to be completed?

  Answer: *One complete revolution around the circle, or 2π radians*

7. Discuss the cyclical nature of the sine function.
Tell students that the change in angle measure (or time, when dealing with modeling sound waves) that it takes to complete one cycle is called the *period* of the function. Graphically, this translates into the horizontal width of one full wave between consecutive crests.

Ask:

- How are the graphs of sine and cosine function alike?

  Possible answer: *They are both waves that reach a maximum value of 1 and a minimum value of –1. They have the same period—it takes 2 radians for both to complete one full cycle.*

- How are the graphs of sine and cosine function different?

  Possible answer: *They have a different starting value. At 0 radians, the value of sine is 0, while at 0 radians the value of cosine is 1, the maximum value.*

- What is the period of the cosine wave you generated?

  Possible answer: *The period is also 2π.*

Tell students that both the sine and the cosine functions could be used to model sound. They will use the sine function in later activities.

Note: In subsequent activities, students build on and transform the graphs of these parent functions to understand how changes in the wave equations affect the corresponding sound.

Handout 3:
Finding the Wave Function

The steps below will help you find a mathematical function used to represent sound waves. Follow the instructions and write down your answers to each question.

Step 1: Traveling a circle

Look at the waveform applet, which shows a circle on a coordinate grid. At the bottom of the applet you'll see a line and a dot labeled "time = 0."

- Click on the time dot and slowly drag it to the right, so time increases from 0 to 10. Notice that the red dot travels around the circle.
- At what time value does the red dot return to its original position on the circle?

  Answer: 6.3

- Imagine that you can keep increasing the time, past time = 10. When do you think the red dot would return again to its original position?

  Answer: 12.6

  Note: Answers may vary. Any time that is a multiple of approximately 6.3 is correct; the point is for students to recognize that the dot *will* return at a later time. If they understand the concept of the periodic (cyclical) nature of this process, they will realize that the dot will return to its original position at multiples of approximately 6.3.

The red dot's travel around the circle is *cyclical*, just as a wave is cyclical, because it keeps repeating its path, always returning to where it's been before. Circles and wheels are often used (in engines, for example) to create an ongoing, cyclical behavior.

Step 2: Graphing the wave

Stay with the waveform applet, returning the time dot to the time = 0 position.

- Click the box next to "Graph the wave" so a check appears in the box.
- Once again, click and slowly drag the time dot to the right. This time the circle will move so that the dot's *x* value is always the same as the time value. The dot will leave images of itself as it moves.
- At what time does the red dot hit the first *crest* (the high point of the wave)?

  Answer: 1.6

- Where is the dot at about twice that time (twice the time it took to hit the first crest)?

  Answer: At 3.2, the dot is at (3.2, 0).

- Where is the dot at about three times the time it took to hit the first crest?

  Answer: At 4.8, the dot is close to the first trough, around (4.8, –1). (It's actually closer to the first trough at time = 4.7.)

- Where is the dot at about four times the time it took to hit the first crest?

  Answer: At 6.4, the dot is just past its original position, with a y value of just over 0.

The wave created by the dot traveling around the circle looks a lot like a sound wave. If you could keep moving the time dot past 10, it would continue to make the same wave over and over again.

Step 3: Finding a function

Now that you can see a way to create a graph that looks like the wave, try to find a mathematical function that can be used to represent that graph. Stay with the waveform applet, keeping the box next to "Graph the wave" checked.

- Click the box next to "Show height" so that a check appears in the box.
- Move the time dot so that you can clearly see a right triangle inside the circle. (The exact time value doesn't matter, just find one for which you can comfortably see the triangle.)
- Sketch the circle and triangle below, just as they appear on your screen.

  Answers will vary. Student sketches should show a circle with a right triangle inside so that the hypotenuse is a radius of the circle and the legs are horizontal and vertical. The hypotenuse should be labeled with a length of 1.

- What is the length of the hypotenuse? Label your sketch with that length.
- The height of the triangle is the *y* value of the red dot—that is, it's the value of the function for that time value. With your partner, consider what functions you might know from past work that express ratios comparing the sides of right triangles. (Hint: The functions use an *angle* in the triangle.) Be prepared to discuss your ideas about what functions are helpful.

  Possible Answer: The trigonometric ratios of sine, cosine, and tangent relate the measures of the acute angles in a right triangle to the ratios of the sides of the triangle. If the radius of the circle is 1 unit, then when the red dot is in the first quadrant, the sine of the central angle is y, the height of the triangle.

Think about ways to connect the *angle* of the triangle at the center of the circle with the *distance* the dot has traveled by answering the following questions:

- How are the angles in the functions that express ratios comparing the sides of right triangles related to the angle at the center of the circle in the applet?

  Possible Answer: The acute angles used in the (trigonometric) functions in a right triangle are the same as the acute angle between the ray from the center of the circle through the red dot and the x-axis.

- How is the angle of the triangle at the center of the circle connected to the distance traveled by the dot around the circle in the applet?

  ***Possible Answer:** As the angle of the triangle at the center of the circle increases, the dot moves, tracing the circumference. If the radius of the circle is 1 unit, when the angle is 360°, the distance traveled by the dot is 2π units.*

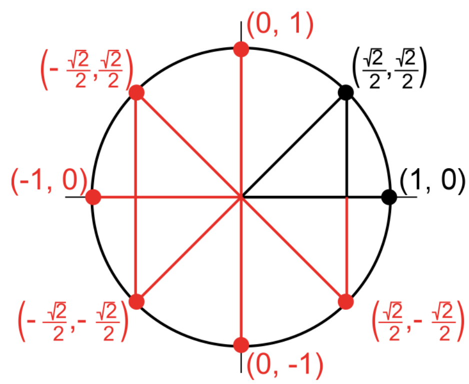
Handout 4:
The Graph of Cosine

As you've seen, the sine and cosine functions can be used to model sound waves. To help you better understand the cyclical nature of these functions, you will generate the graph of the cosine function using right triangles and a unit circle.

Part 1: Generating Cosine

- Start by drawing a right triangle in the unit circle shown below. The unit circle has radius 1. In the diagram, a 45°–45°–90° triangle has been drawn in the unit circle, with the vertex of one of the acute angles at the center of the circle.

Answer:



- Following this example, use a protractor to draw right triangles inside the unit circle, measuring angles in the counterclockwise direction. Use an increment of 45° as you increase angle measures. Note that at 0°, 90°, 180°, 270°, and 360°, you will be unable to sketch right triangles. For these angles, mark the coordinates of the point on the circle (e.g., at 0°, the coordinates of the point on the circle where the initial side of the triangle and the terminal side of the triangle coincide are [1, 0]).
- Complete the Table of Cosine Values below, using the definition of the cosine of an angle, the ratio of the adjacent side of the angle, and the hypotenuse of the right triangle. (The first entry is already completed.)

Notes:

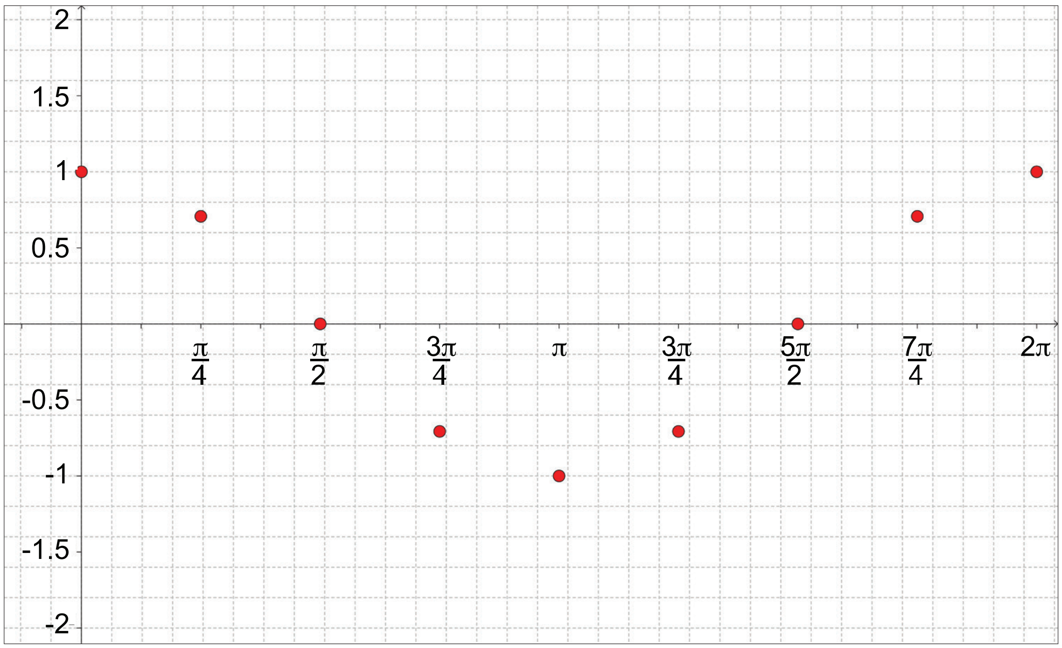
- *The position of the angle drawn in the unit circle matters when determining the* ***sign*** *of the value of the cosine of the angle.*
- *Use your knowledge of special triangles to determine the cosine of multiples of 45°.*
- *Use the* ***symmetry*** *of the circle to determine the cosine of multiples of 45°.*

Table of Cosine Values

Acute angle measure, ϕ (in degrees)	Acute angle measure, ϕ (in radians)	Cos ϕ	Coordinates of the point on the unit circle
0°	0 radians	1	(1, 0)
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
90°	$\frac{\pi}{2}$	0	$(0, -1)$
135°	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
180°	π	1	$(-1, 0)$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
270°	$\frac{3\pi}{2}$	0	$(0, -1)$
315°	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$
360°	2π	1	$(1, 0)$

Once you have completed the Table of Cosine Values, transfer these values to the coordinate system below. Notice that the x-axis is labeled using radian measure and that the y-axis values range from –1 to 1.

Answer:



Part 2: Discussion Questions

In class, you looked at a graph of the basic sine wave. In Part 1 of this handout, you generated the basic cosine wave from right triangles drawn inside a unit circle. Reflect on your work by answering the following questions:

1. What are the similarities and the differences between the graphs of $y = \sin(x)$ and $y = \cos(x)$?

   Answers will vary. Similarities might include the range of each (from –1 to 1) and the possibility for repetition if additional angles are added beyond one rotation of the circle. Differences might include the coordinates of the point on the circle and their correspondence to either the sine or the cosine value of the acute angle of the right triangle.

2. Why does each graph range between the values –1 and 1?

   Answer: The right triangles were drawn in a unit circle, which means that the greatest y value is 1 and the least is –1. Since sine measures the height of points on the unit circle, the y values are the sine values. Similarly, the x values are the cosine values, and the x values on the unit circle are all between –1 and 1.

3. How might you change the exercise above to create a graph of a sine wave that ranges between –2 and 2? Try doing this on a separate piece of graph paper.

   Answer: A circle with a radius of 2 will generate stretched sine and cosine waves. You could also multiply all the y values by 2.

4. How could you generate additional values that would yield a closer approximation to a waveform?

   Answer: Consider drawing additional acute right triangles. For example, just within the first quadrant, two new points on the wave can be generated by adding an acute right triangle with the 30° angle at the vertex and another one with the 60° angle at the vertex. Then, use the symmetry of the circle to reflect the right triangles into the other quadrants. Measure the angles from the 0 position in a clockwise direction to the hypotenuse of the right triangle, and use the height of the triangle to obtain other points.

5. Did generating the basic cosine wave using the unit circle lead you to any new ideas or discoveries?

   Answers will vary.

Part 2: Making Changes

Students work with transformations of functions to determine the effect of multipliers and added constants on the graphs of the functions. They start with algebraic functions and then extend their conclusions to the sine function.

Length

3 50-minute sessions

Activity 2A: Transforming Functions

To prepare for the kinds of changes they will apply to the equations in Activity 2B, students revisit the graphs and equations of known functions and explore how changes in the coefficients of a function's equation affect its graph. Types of changes explored include horizontal shifts (also called *phase shifts* with trigonometric functions), horizontal stretches (period adjustments), vertical stretches (changes in amplitude), and vertical shifts.

> **Note:** If students have worked on transforming functions in these ways before, you may choose to skip this activity.

Understandings

- Adding a constant to a function shifts the graph up if the constant is positive and down if the constant is negative.
- Multiplying a function by a constant stretches the graph vertically if the absolute value of the constant is greater than 1 and compresses it vertically if the absolute value of the constant is less than 1.
- Multiplying a function by a negative constant flips the graph vertically across the *x*-axis.
- Subtracting a constant from the independent variable in a function shifts the graph to the right if the constant is positive and to the left if the constant is negative.
- Multiplying the independent variable in a function stretches the graph horizontally if the absolute value of the constant is greater than 1 and compresses it horizontally if the absolute value of the constant is less than 1.

> **Note:** If you share these understandings with students, point out that some of them may seem counterintuitive—for example, when a positive constant is subtracted from the independent variable, you might expect the graph to move to the left, but instead it moves right; when the multiplier of the independent variable is greater than 1, you might expect the graph to stretch, but instead it shrinks horizontally. It may be helpful to point out that when the parameter is applied to the function, it affects the graph vertically in an intuitive manner, but when it is applied to the independent variable, it affects the graph horizontally in the opposite way from what you might expect.

Materials Needed

- **Handout 5A–D: Function Transformations Group 1–4** (one copy of each part for one-fourth of the class)
- Graphing calculators
- Graph paper

1. Have students transform functions.
Divide the class into four Function Groups, and assign each group one of the four functions: $y = x^2$, $y = x^3$, $y = 3^x$, and $y = \left(\frac{1}{3}\right)^x$.

Give each Function Group member the appropriate version of **Handout 5A–D: Function Transformations Group 1–4**, a graphing calculator, and graph paper. Have each student complete Step 1 of Handout 5 and then one of the problems in Step 2.

> **Note:** Each version of Handout 5 has four problems in Step 2. You can assign a problem to each group member or have groups assign problems to their members. Be sure that each group completes all four problems and that each group member completes at least one problem.

Have Function Group members check one another's work after completing the problems in Part 2.

2. Have students meet in new groups to share their work on Handout 5.
Re-organize students into four Transformation Groups, consisting of all the students who performed the *same problem* (that is, the same transformation) in Part 2. Students from different Function Groups will have used different parent functions, so each group can see that the effect of the transformation is independent of the parent function. Have Transformation Groups discuss the similarities and differences between each group member's graphs.

3. Have students display the graphs of their transformations.
Distribute chart paper and markers to each Transformation Group. Have each group draw and display their graphs, showing the same transformation applied to different parent functions.

4. Have each Transformation Group share its findings.
Have one or more students from each Transformation Group explain the similarities in the equations and graphs of the group's different functions. Have students note the effects of each type of transformation on their handouts. Keep the chart paper posted so that students can see the parallels between these transformations and the ones they'll later perform on the trigonometric functions.

As groups present their work, let them use their own informal language to describe the effect. You can then identify the formal names of the transformations, as follows, and have students add those names to their chart:

- Problem 1: Vertical shift
- Problem 2: Vertical stretch
- Problem 3: Horizontal shift
- Problem 4: Horizontal stretch

5. Have Function Groups work on Step 3 of Handout 5.

Have students return to their Function Groups to complete Step 3 of Handout 5, which gives two functions with two or three transformations. Have group members work together to use the results of the individual transformations to predict how the graphs will look, and then graph each equation to see how close their predictions were.

Handout 5A:
Function Transformations Group 1

Parent function: $y = x^2$

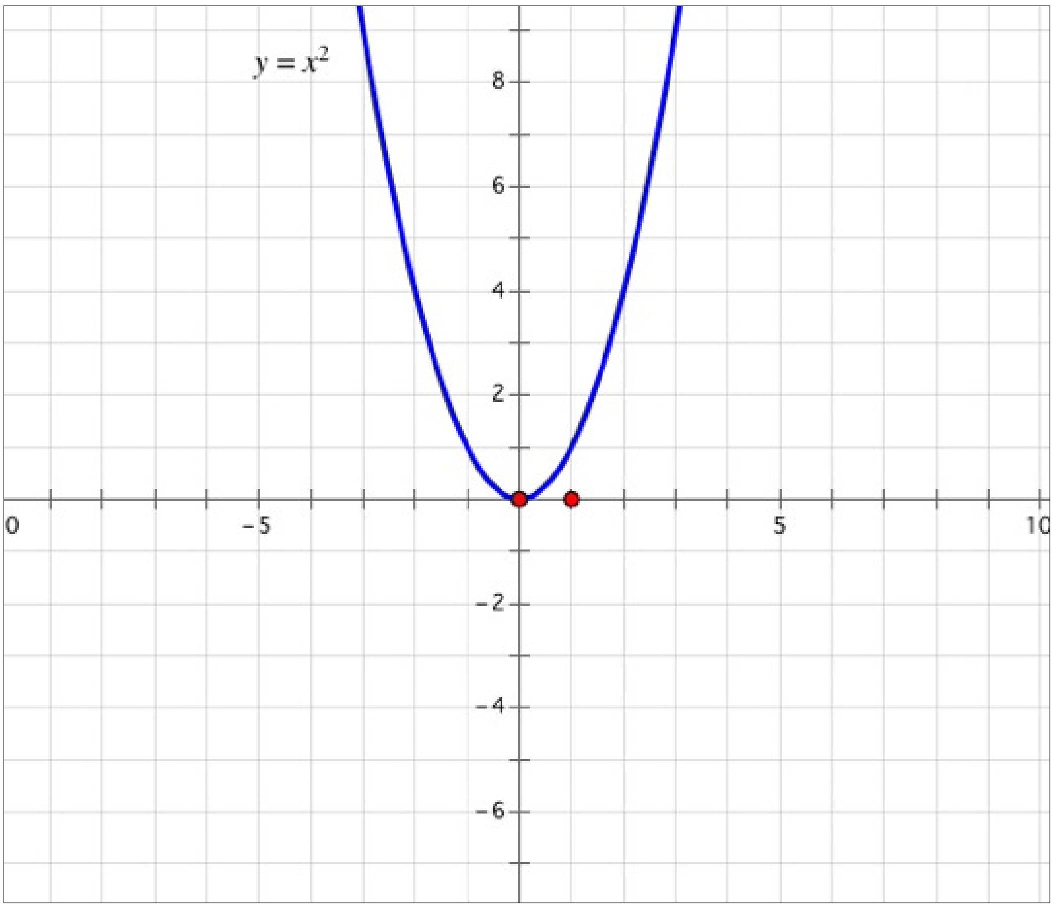
In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you will use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #1, will work on altering the quadratic equation whose graph is the basic parabola. Use separate sheets of graph paper for your work.

Step 1

Each group member should sketch a copy of the function $y = x^2$. (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

Solution:



Step 2

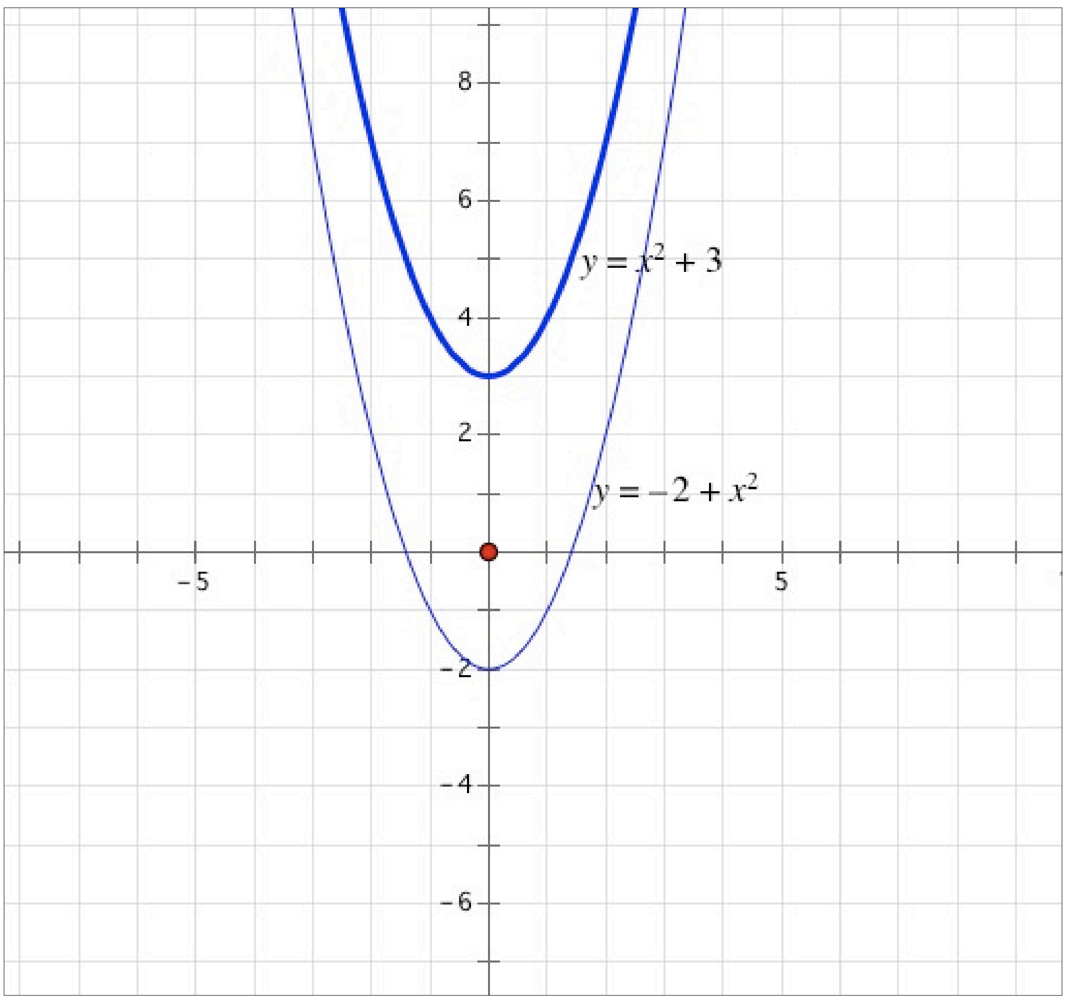
The following problems make four different kinds of changes to the original parent equation of the parabola you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

Problem 1

Graph the following equations on the same axes as the parent function $y = x^2$:

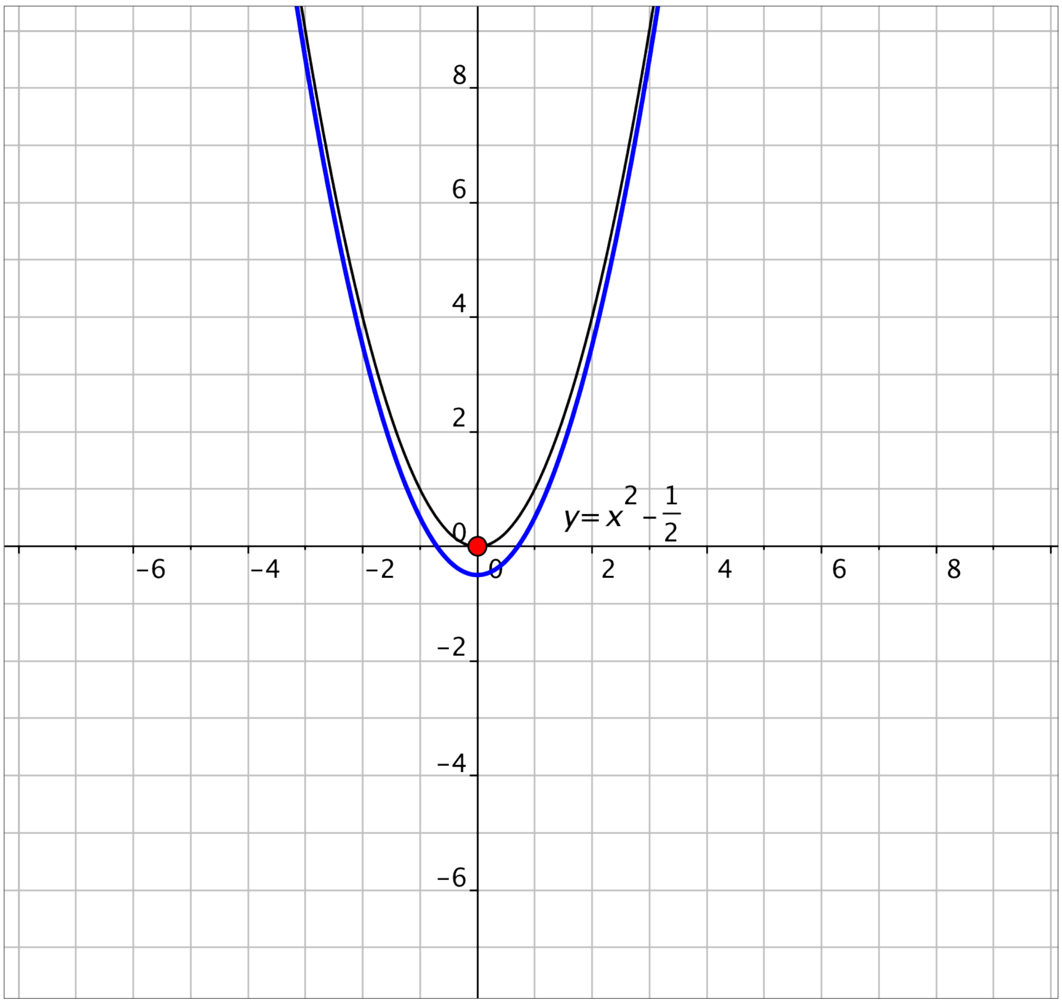
$y = x^2 + 3$ $\qquad$ $y = -2 + x^2$

Solutions:



Generalize your observations by predicting what the graph of $y = x^2 + c$ will look like in comparison to the parent function $y = x^2$, no matter what value c has. Test your idea by predicting what the graph of $y = x^2 - \left(\frac{1}{2}\right)$ will look like, and then graph it.

Solutions:

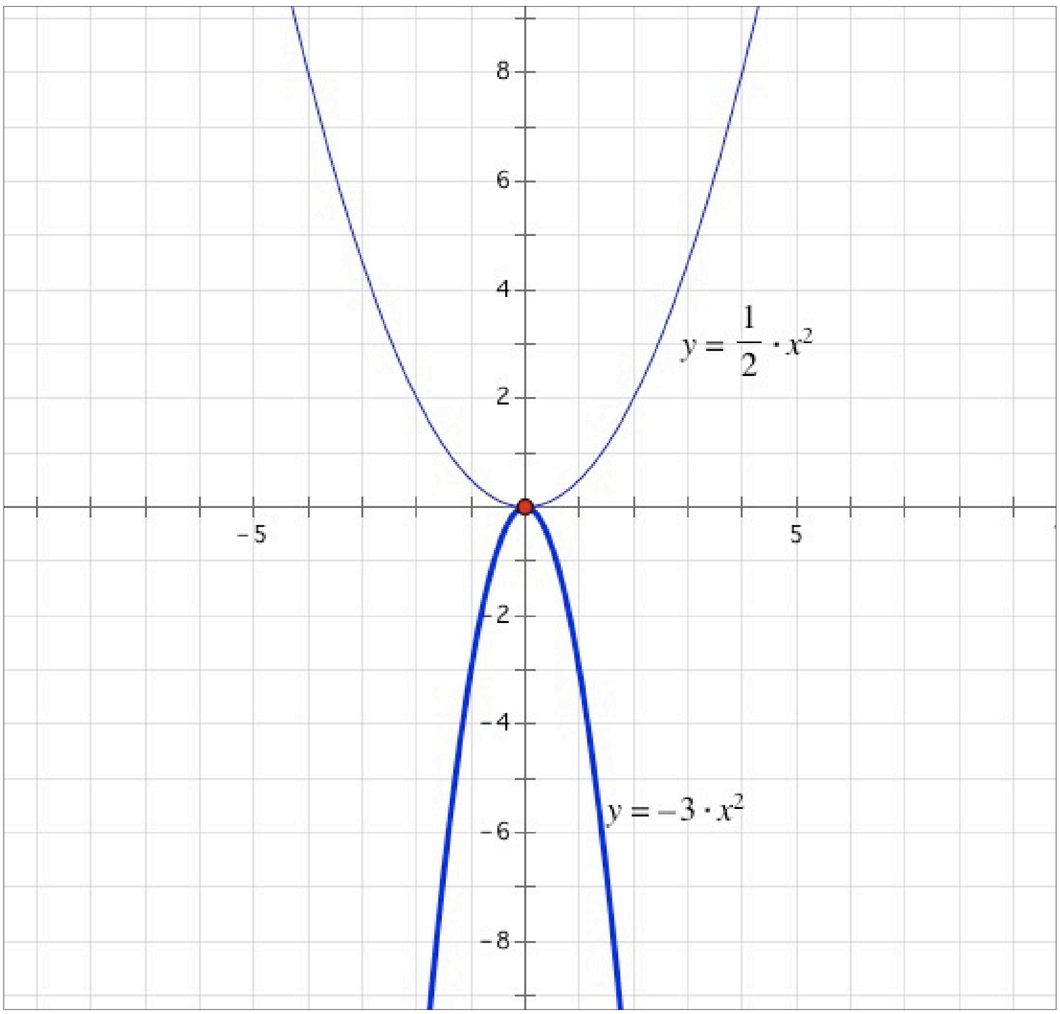


Problem 2

Graph the following equations on the same axes as the parent function $y = x^2$:

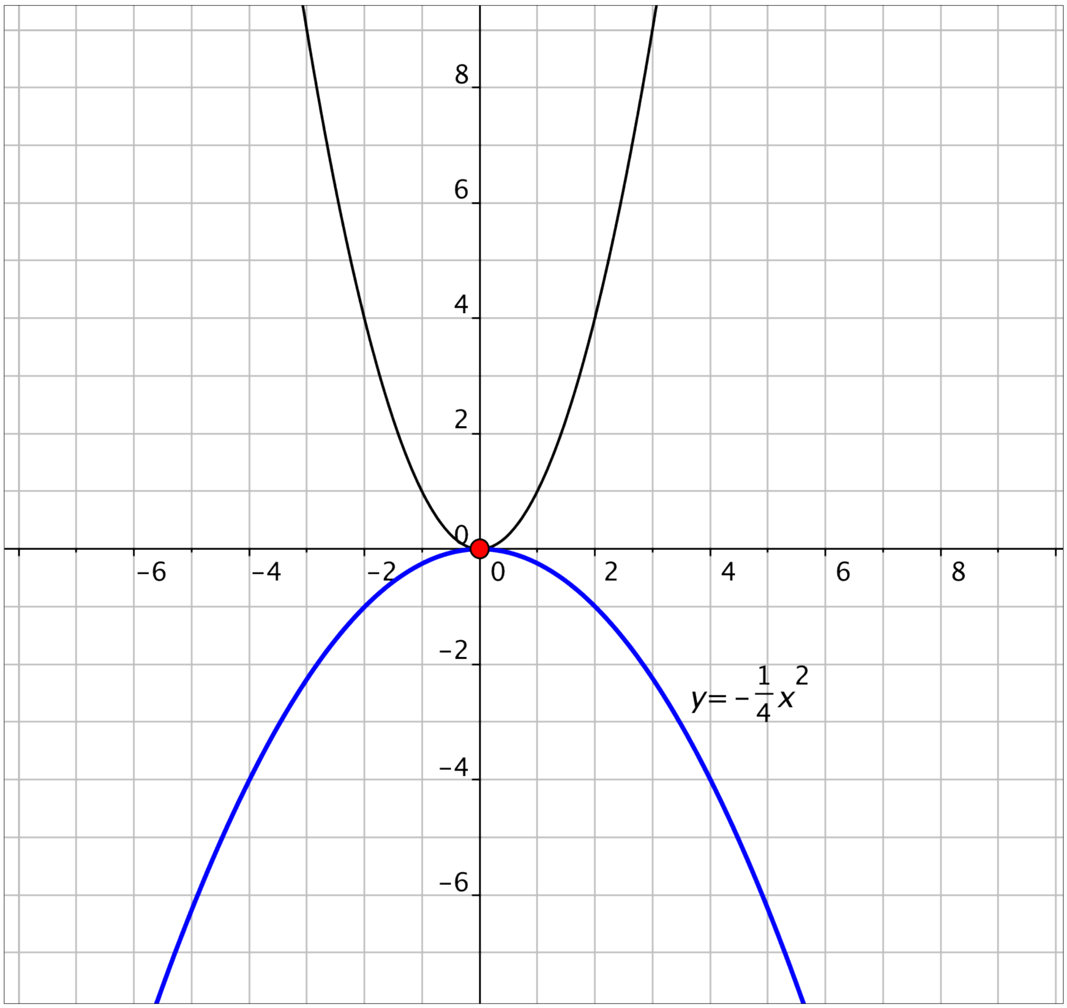
$y = -3x^2$ $\qquad$ $y = \left(\frac{1}{2}\right)x^2$

Solutions:



Generalize your observations by predicting what the graph of $y = cx^2$ will look like in comparison to the parent function $y = x^2$, no matter what value c has. Test your idea by predicting what the graph of $y = -\frac{1}{4}x^2$ will look like, and then graph it.

Solutions:

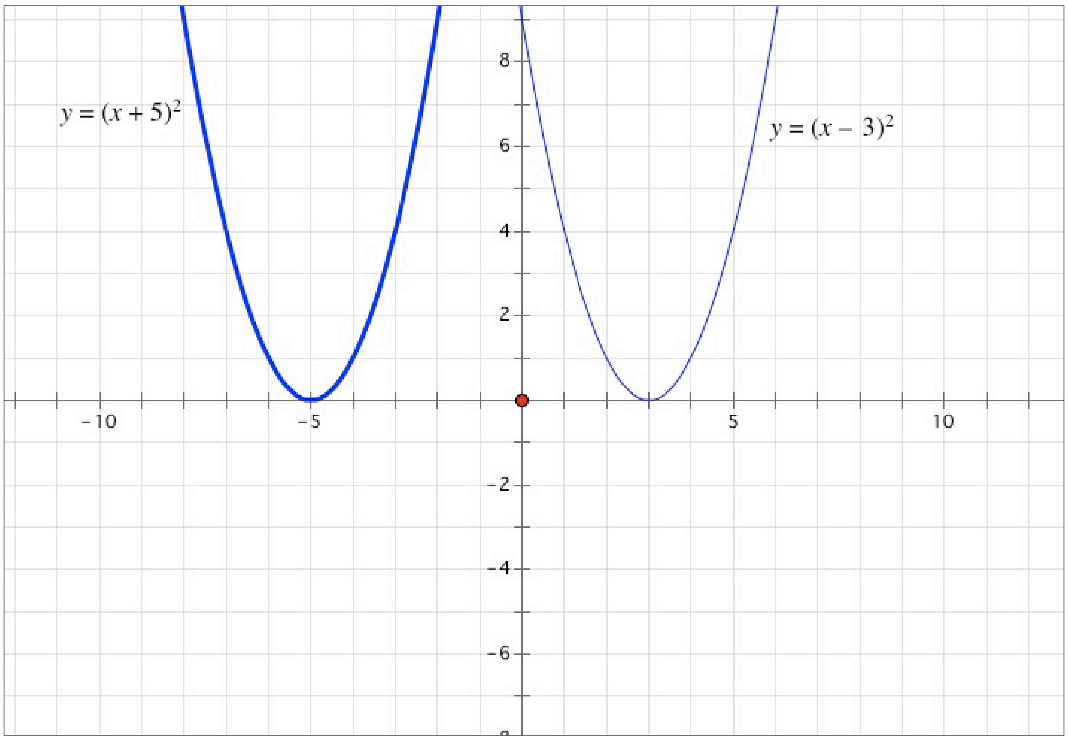


Problem 3

Graph the following equations on the same axes as the parent function $y = x^2$:

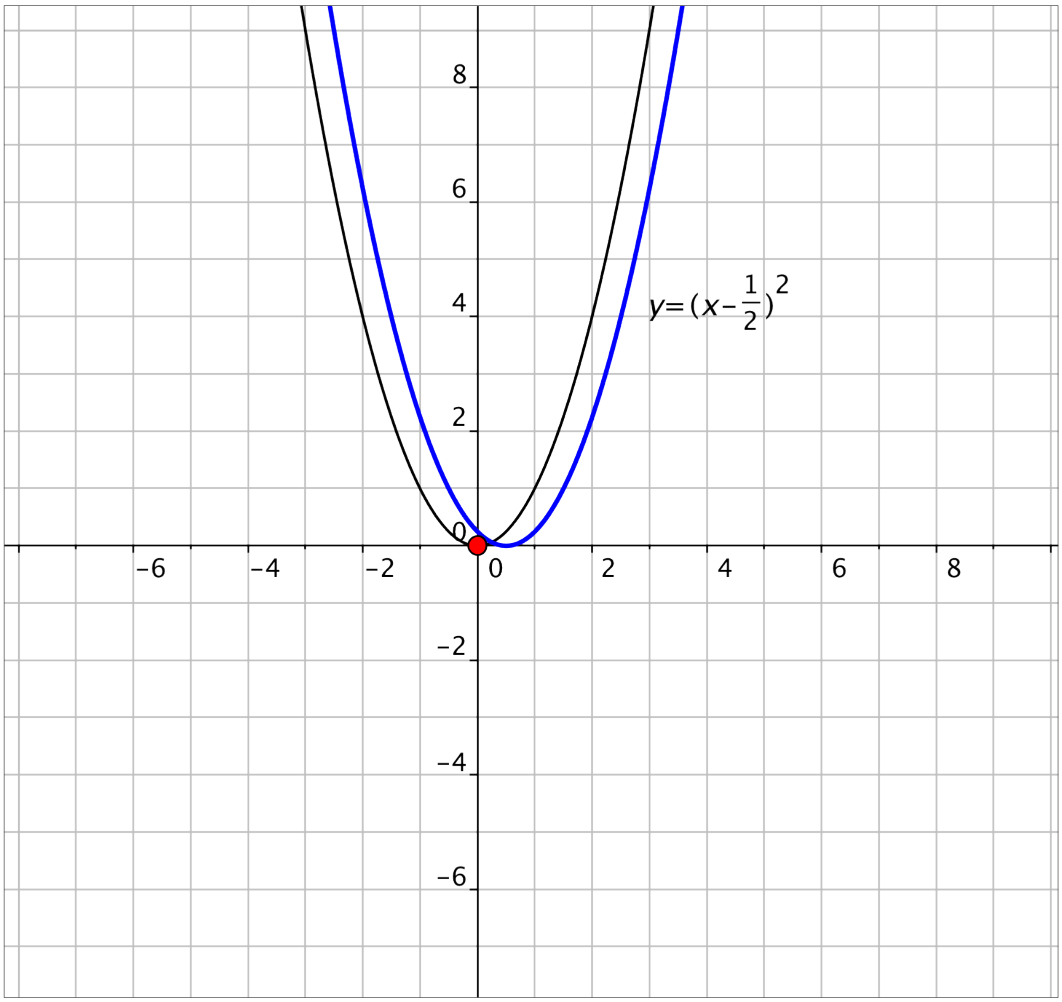
$y = (x + 5)^2$ $y = (x - 3)^2$

Solutions:



Generalize your observations by predicting what the graph of $y = (x - c)^2$ will look like in comparison to the parent function $y = x^2$, no matter what value c has. Test your idea by predicting what the graph of $y = (x - \frac{1}{2})^2$ will look like, and then graph it.

Solutions:

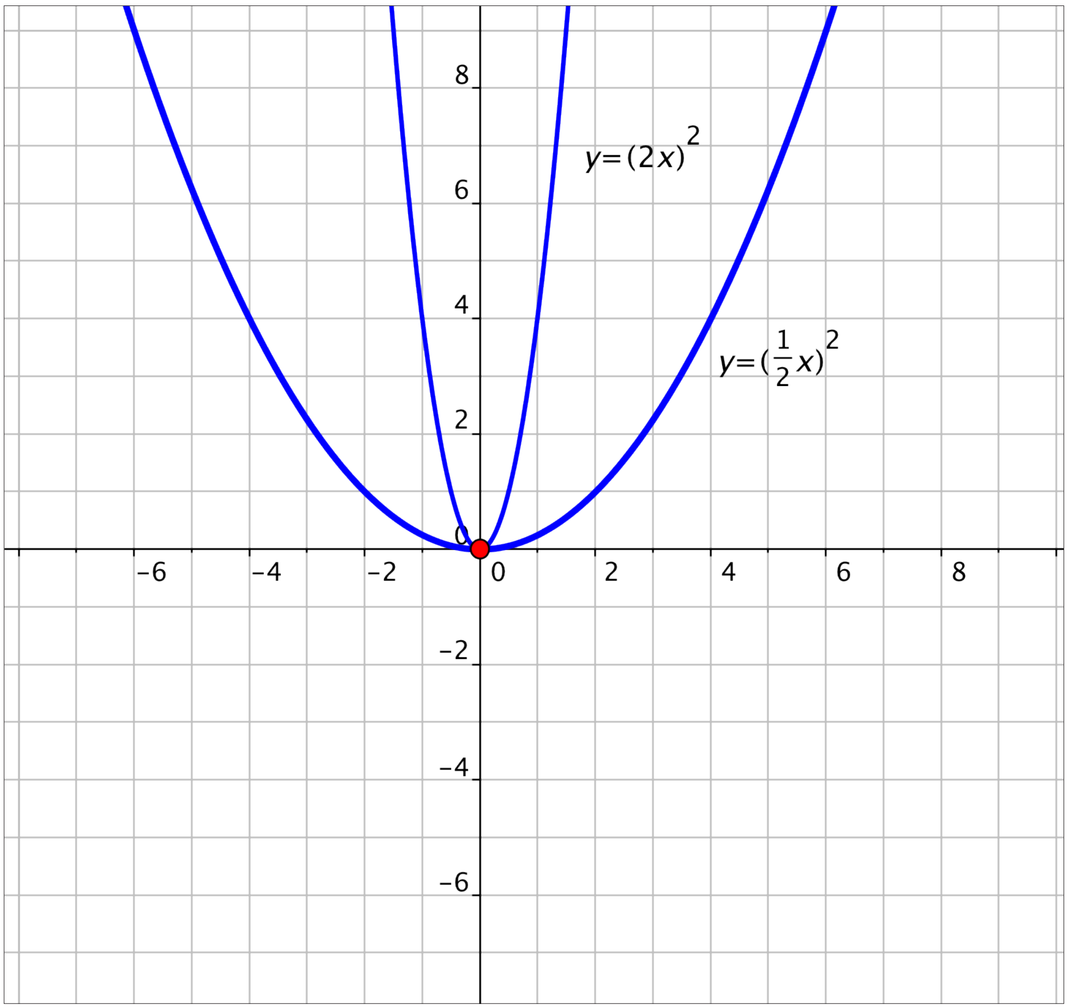


Problem 4

Graph the following equations on the same axes as the parent function $y = x^2$:

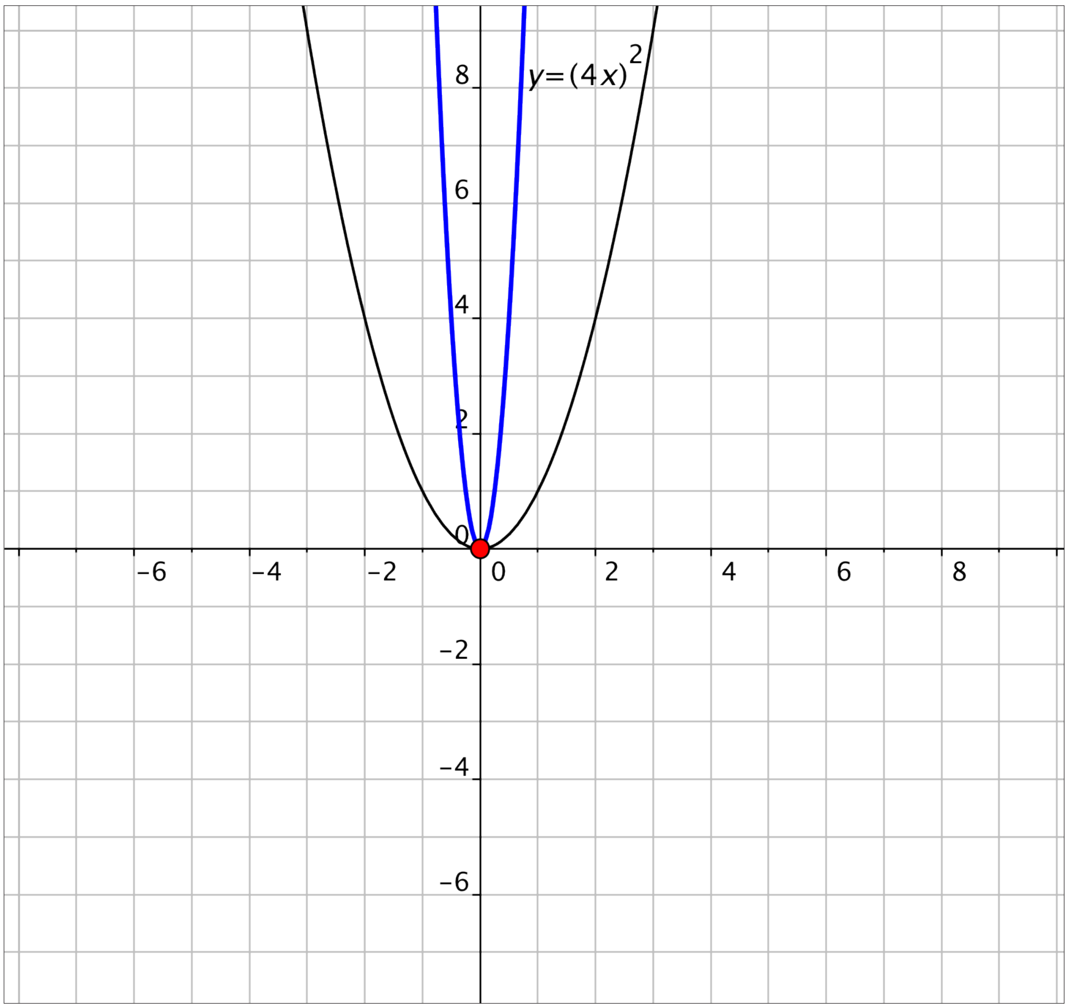
$y = (2x)^2$ $y = (\frac{1}{2}x)^2$

Solutions:



Generalize your observations by predicting what the graph of $y = (cx)^2$ will look like in comparison to the parent function $y = x^2$, no matter what value c has. Test your idea by predicting what the graph of $y = (4x)^2$ will look like, and then graph it.

Solutions:



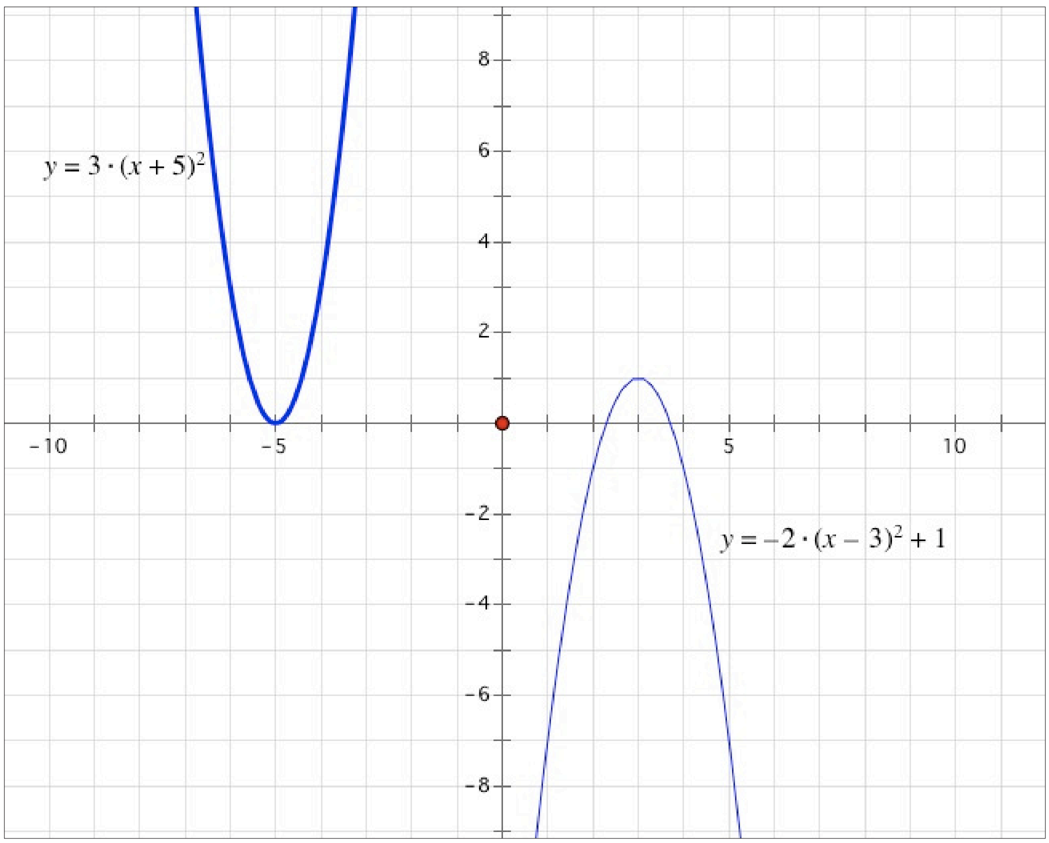
Step 3

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function $y = x^2$:

$y = 3(x + 5)^2$ $y = -2(x - 3)^2 + 1$

After you've recorded your predictions, test them by graphing each equation.

Solutions:



Handout 5B:
Function Transformations Group 2

Parent function: $y = x^3$

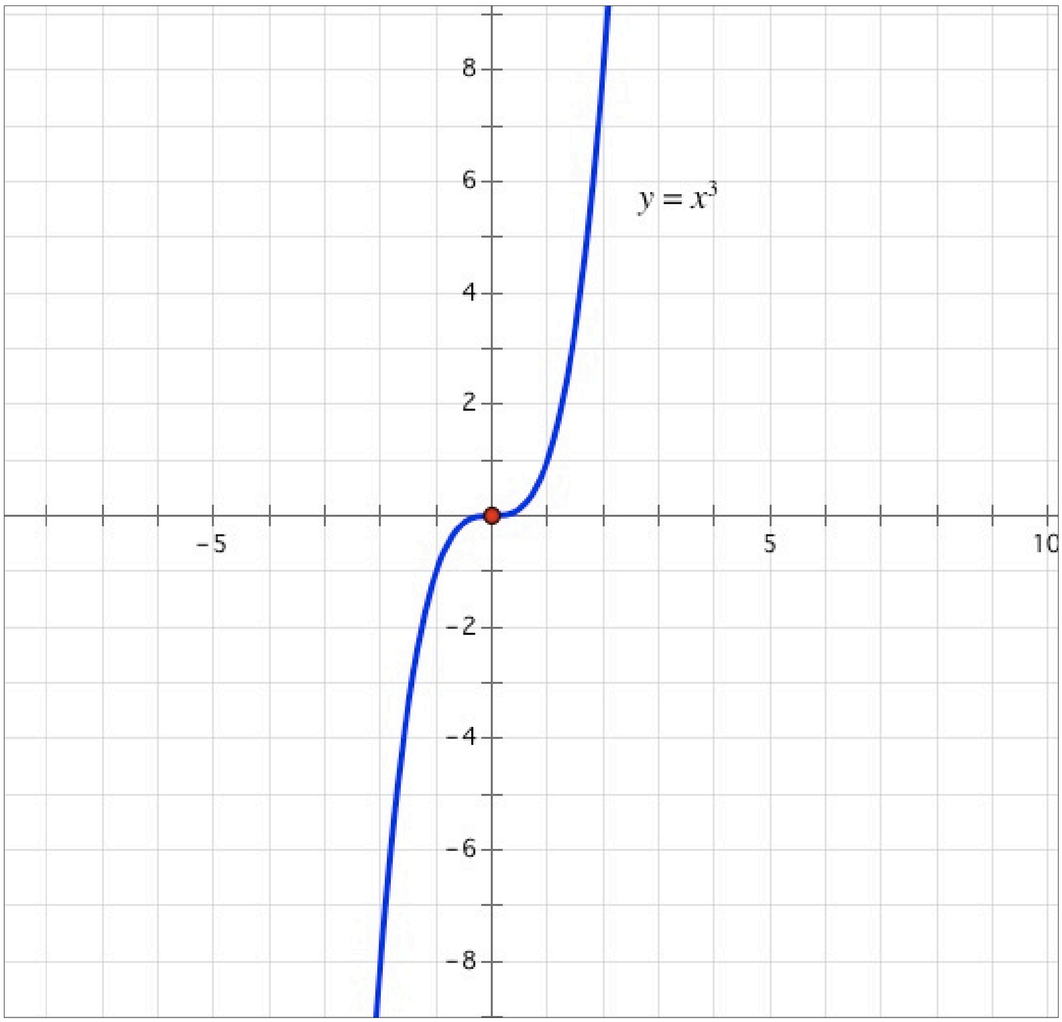
In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #2, will work on altering the quadratic equation whose graph is the basic cubic function. Use separate sheets of graph paper for your work.

Step 1

Each group member should sketch a copy of the function $y = x^3$. (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

Solution:



Step 2

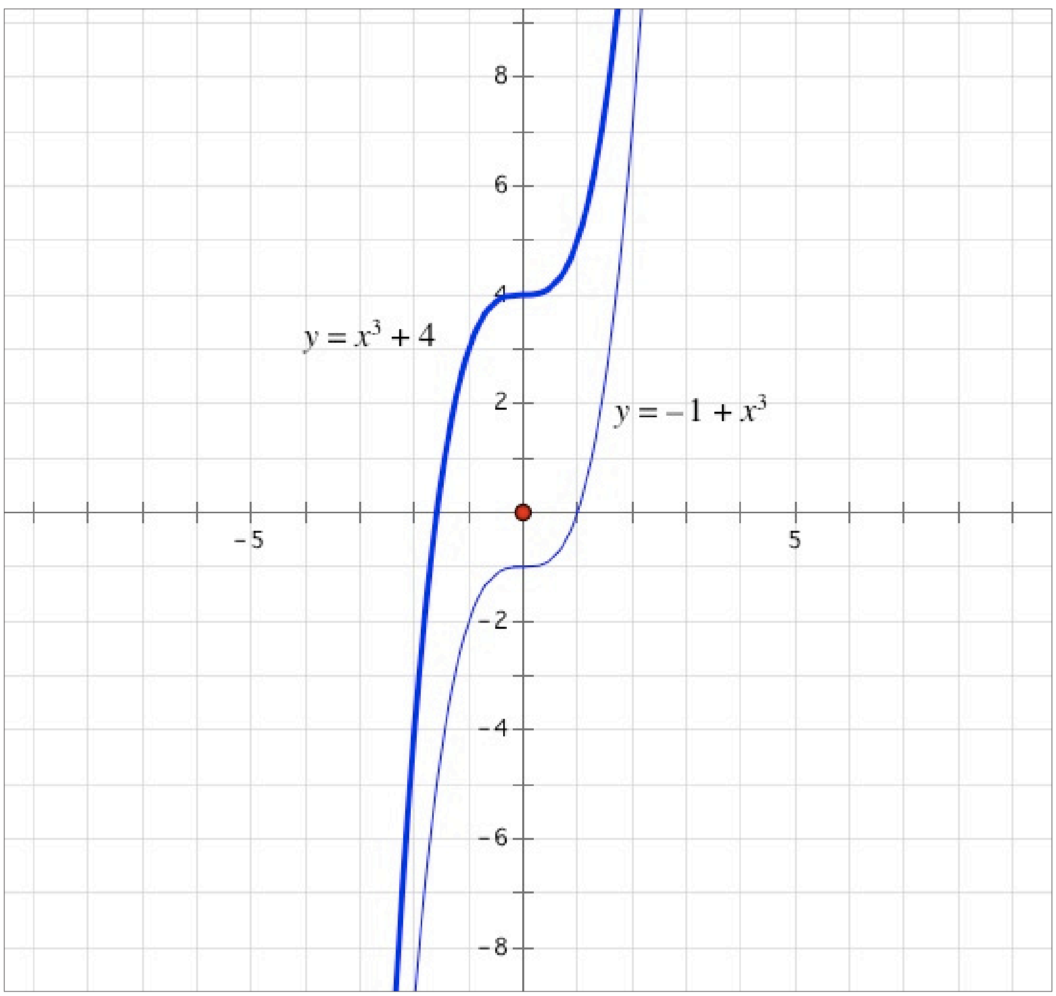
The following problems make four different kinds of changes to the original parent equation of the cubic function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

Problem 1

Graph the following equations on the same axes as the parent function $y = x^3$:

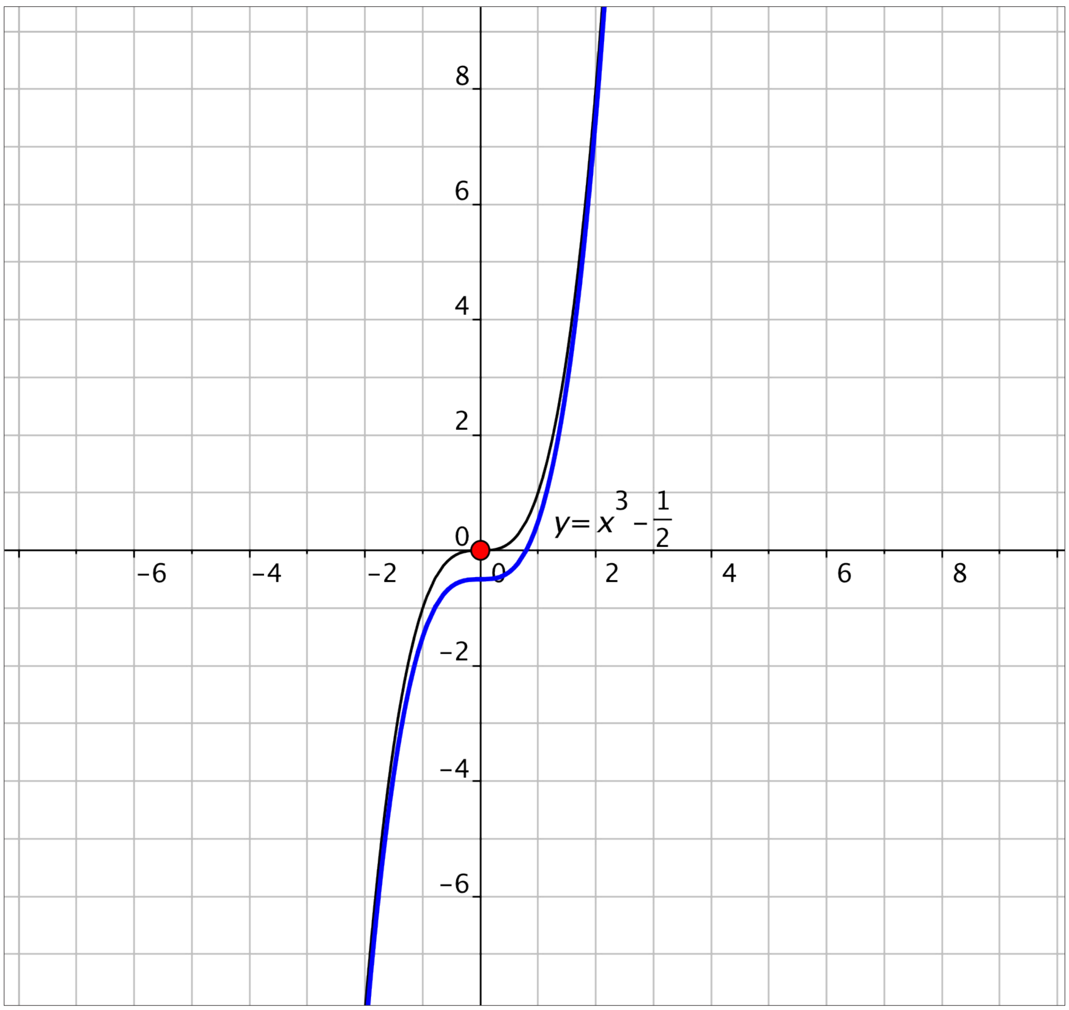
$y = x^3 + 4$ $\qquad$ $y = -1 + x^3$

Solutions:



Generalize your observations by predicting what the graph of $y = x^3 + c$ will look like in comparison to the parent function $y = x^3$, no matter what value c has. Test your idea by predicting what the graph of $y = x^3 - \frac{1}{2}$ will look like, and then graph it.

Solutions:

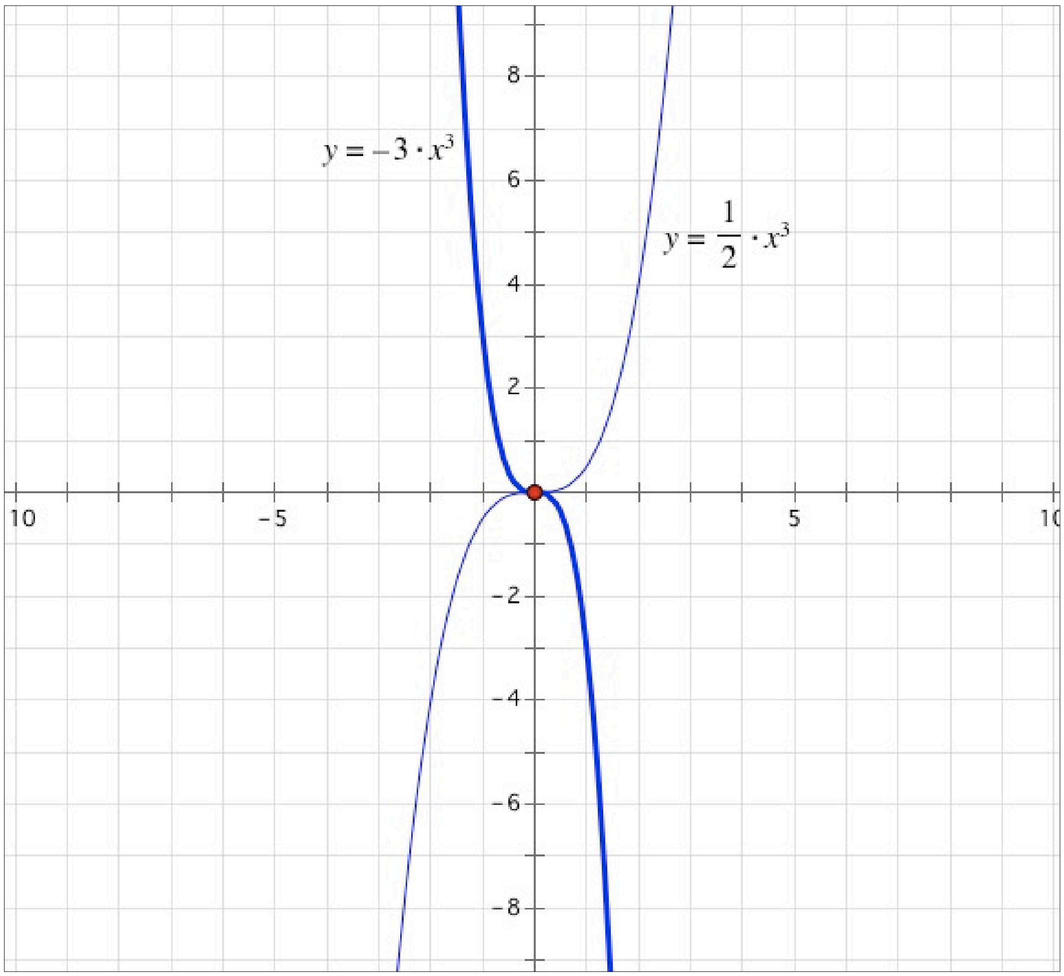


Problem 2

Graph the following equations on the same axes as the parent function $y = x^3$:

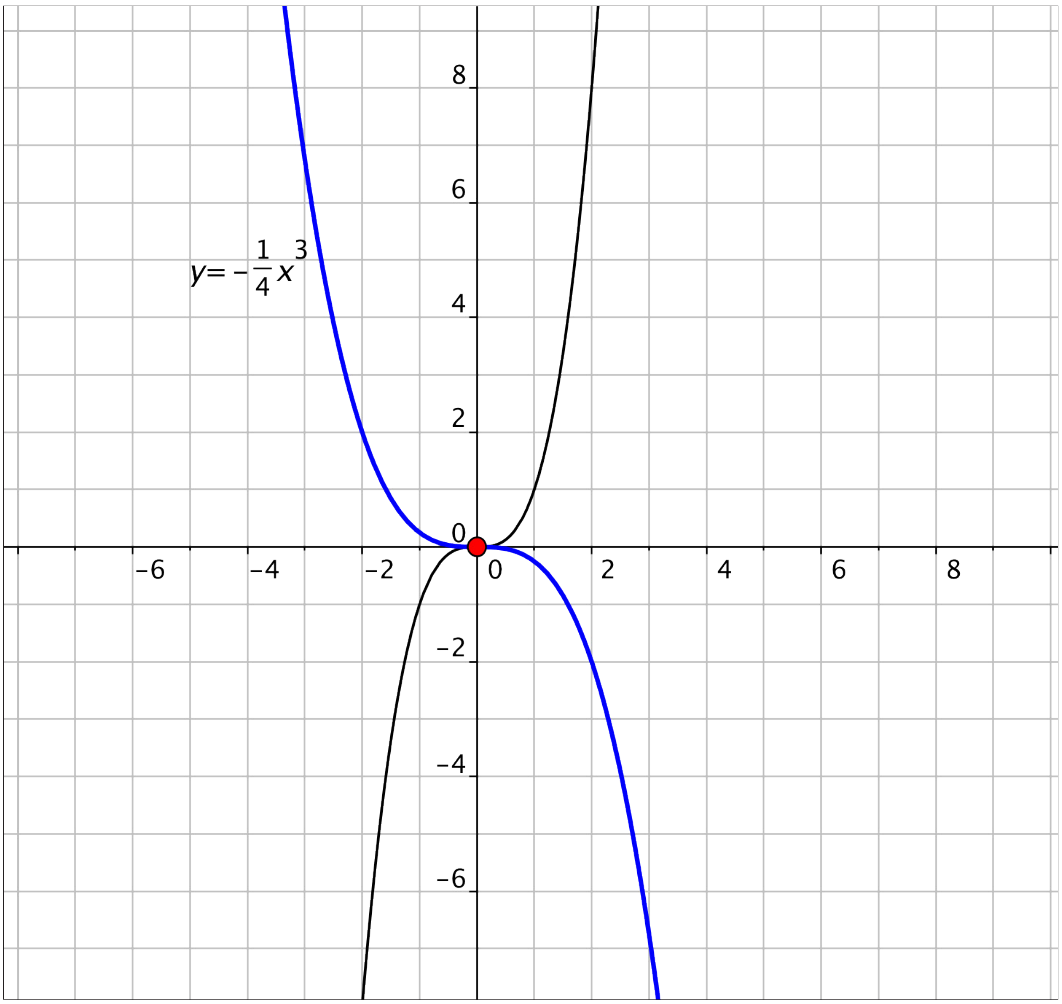
$y = -3x^3$ $\qquad$ $y = \frac{1}{2}x^3$

Solutions:



Generalize your observations by predicting what the graph of $y = cx^3$ will look like in comparison to the parent function $y = x^3$, no matter what value c has. Test your idea by predicting what the graph of $y = -\frac{1}{4}x^3$ will look like, and then graph it.

Solutions:

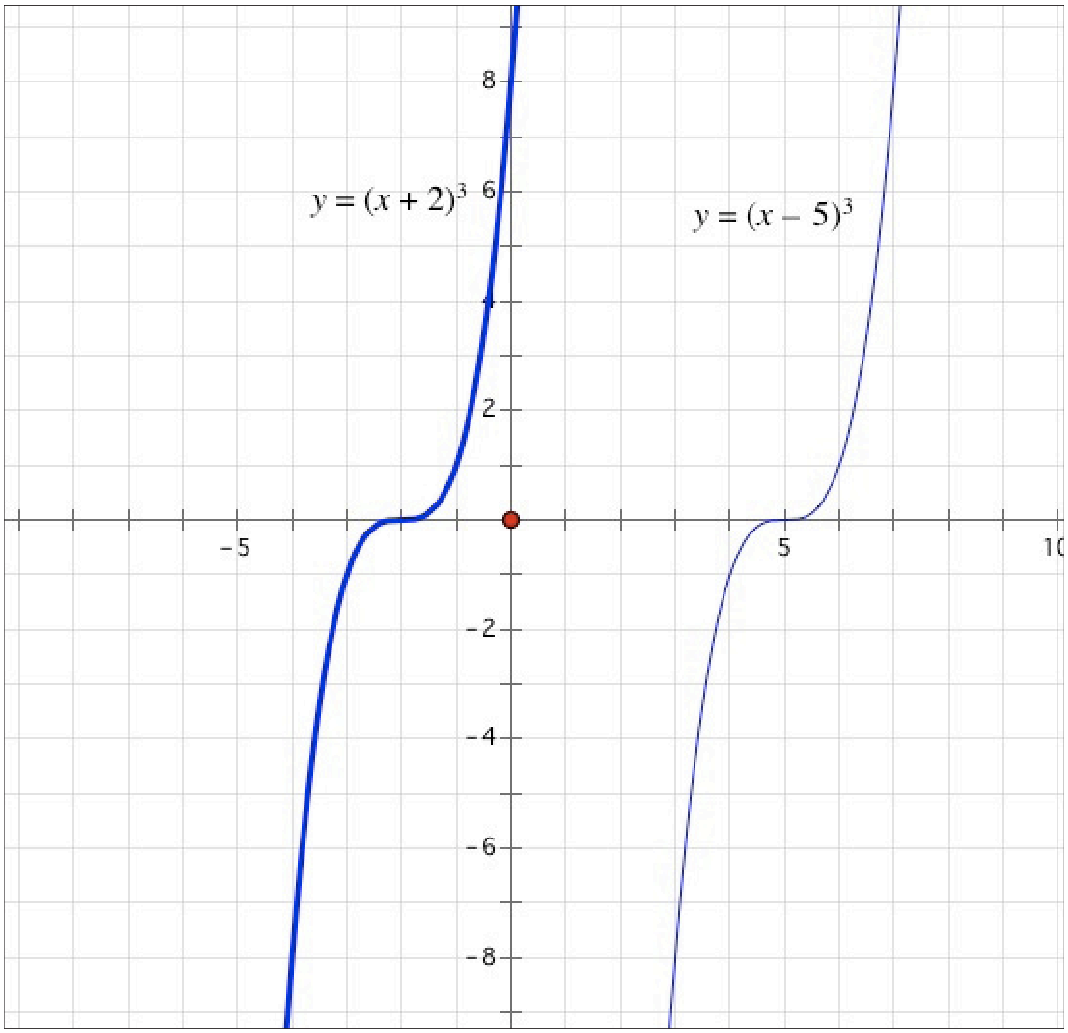


Problem 3

Graph the following equations on the same axes as the parent function $y = x^3$:

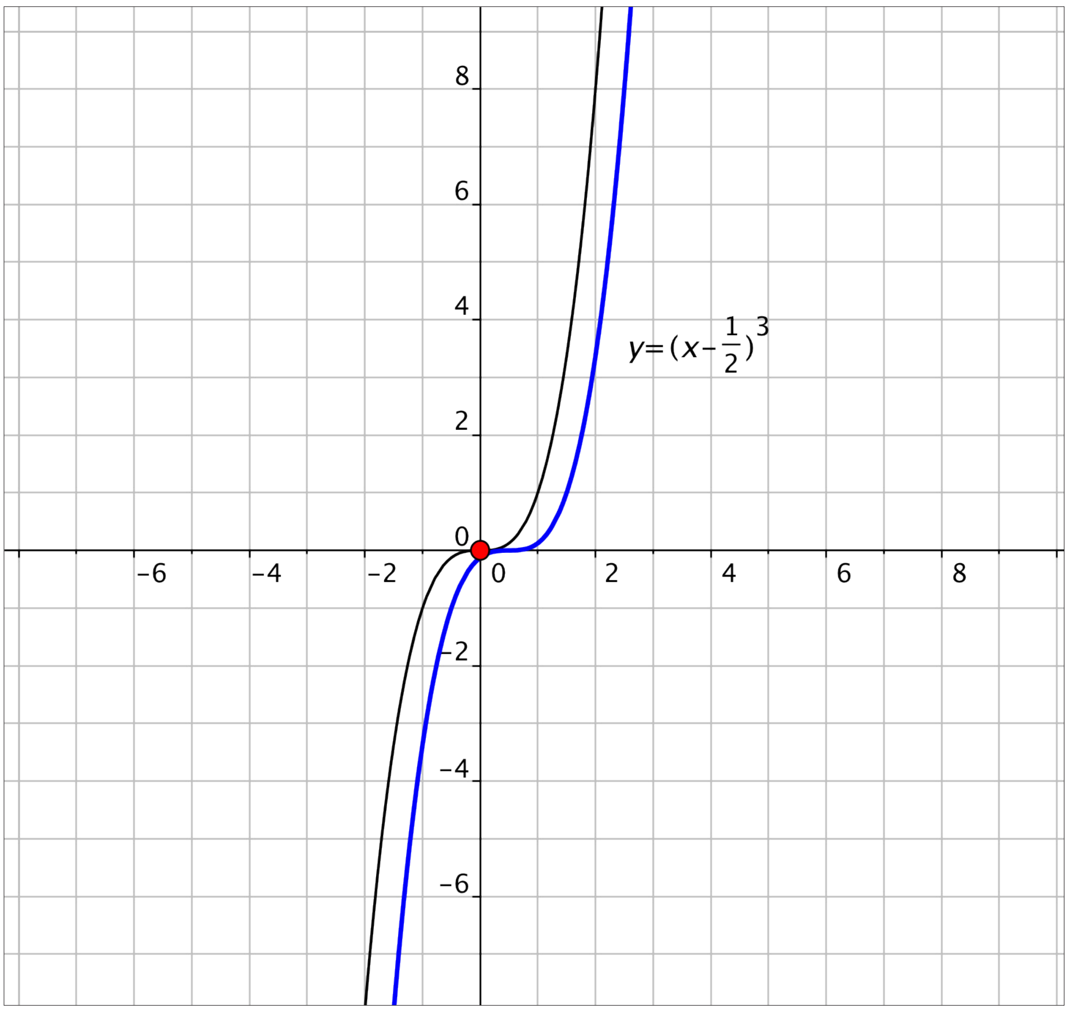
$y = (x + 2)^3$ $y = (x - 5)^3$

Solutions:



Generalize your observations by predicting what the graph of $y = (x - c)^3$ will look like in comparison to the parent function $y = x^3$, no matter what value c has. Test your idea by predicting what the graph of $y = (x - \frac{1}{2})^3$ will look like, and then graph it.

Solutions:

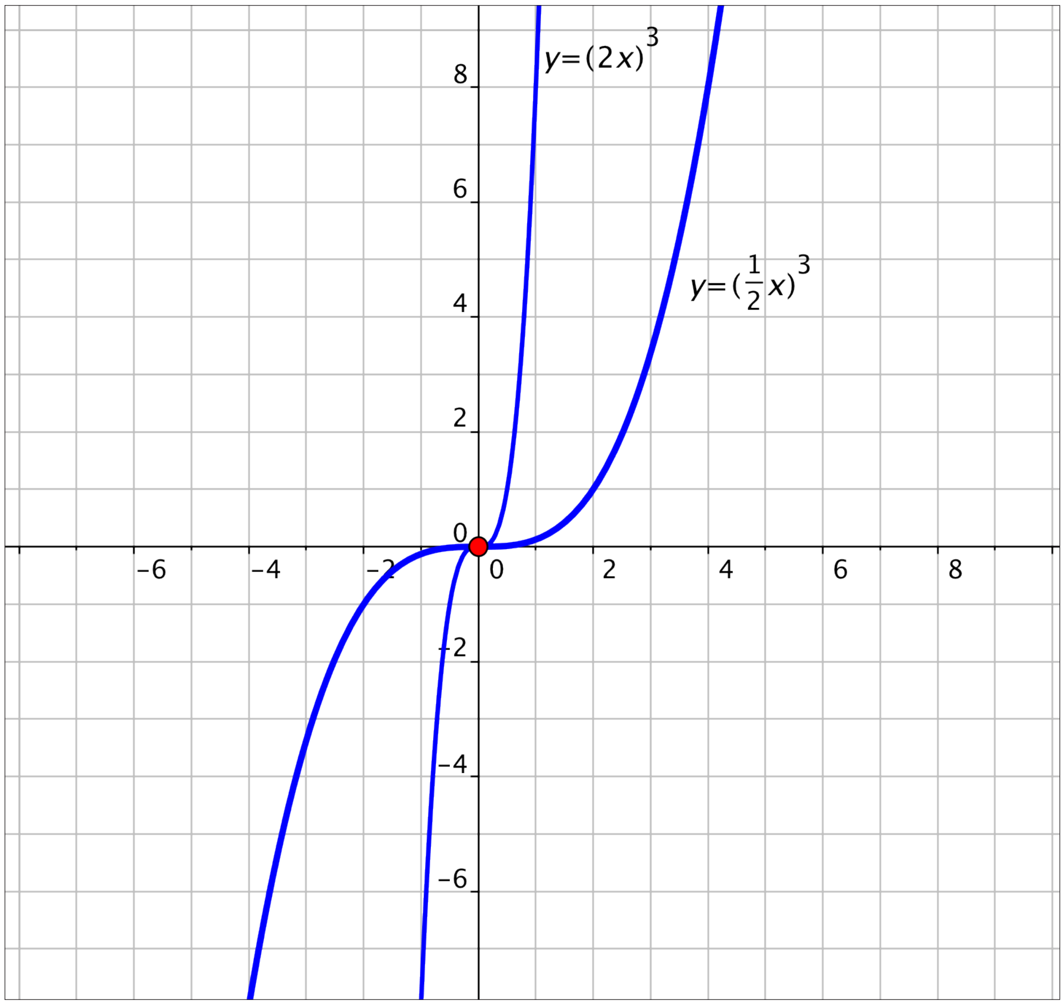


Problem 4

Graph the following equations on the same axes as the parent function $y = x^3$:

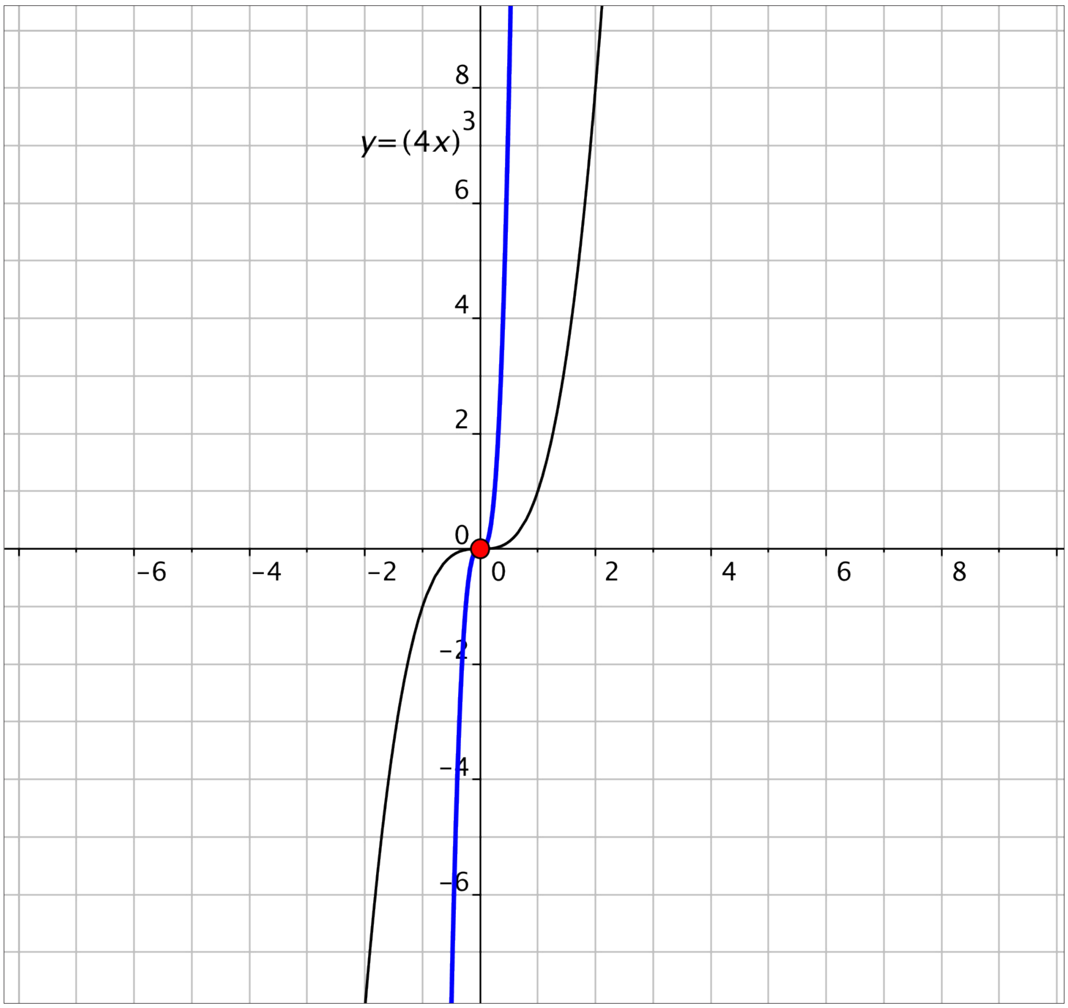
$y = (2x)^3$ $y = (\frac{1}{2}x)^3$

Solutions:



Generalize your observations by predicting what the graph of $y = (cx)^3$ will look like in comparison to the parent function $y = x^3$, no matter what value c has. Test your idea by predicting what the graph of $y = (4x)^3$ will look like, and then graph it.

Solutions:



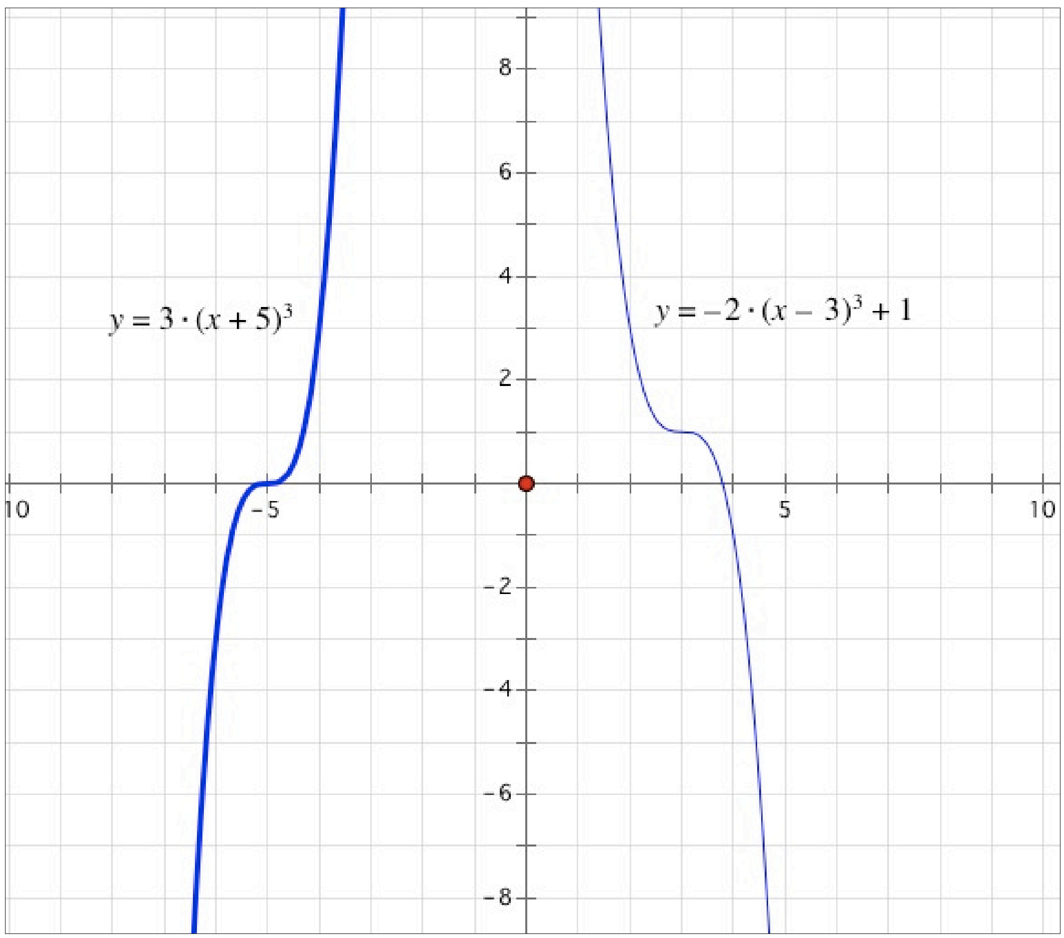
Step 3

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function $y = x^3$:

$y = 3(x + 5)^3$ $\qquad$ $y = -2(x - 3)^3 + 1$

After you've recorded your predictions, test them by graphing each equation.

Solutions:



Handout 5C:
Function Transformations Group 3

Parent function: $y = 3^x$

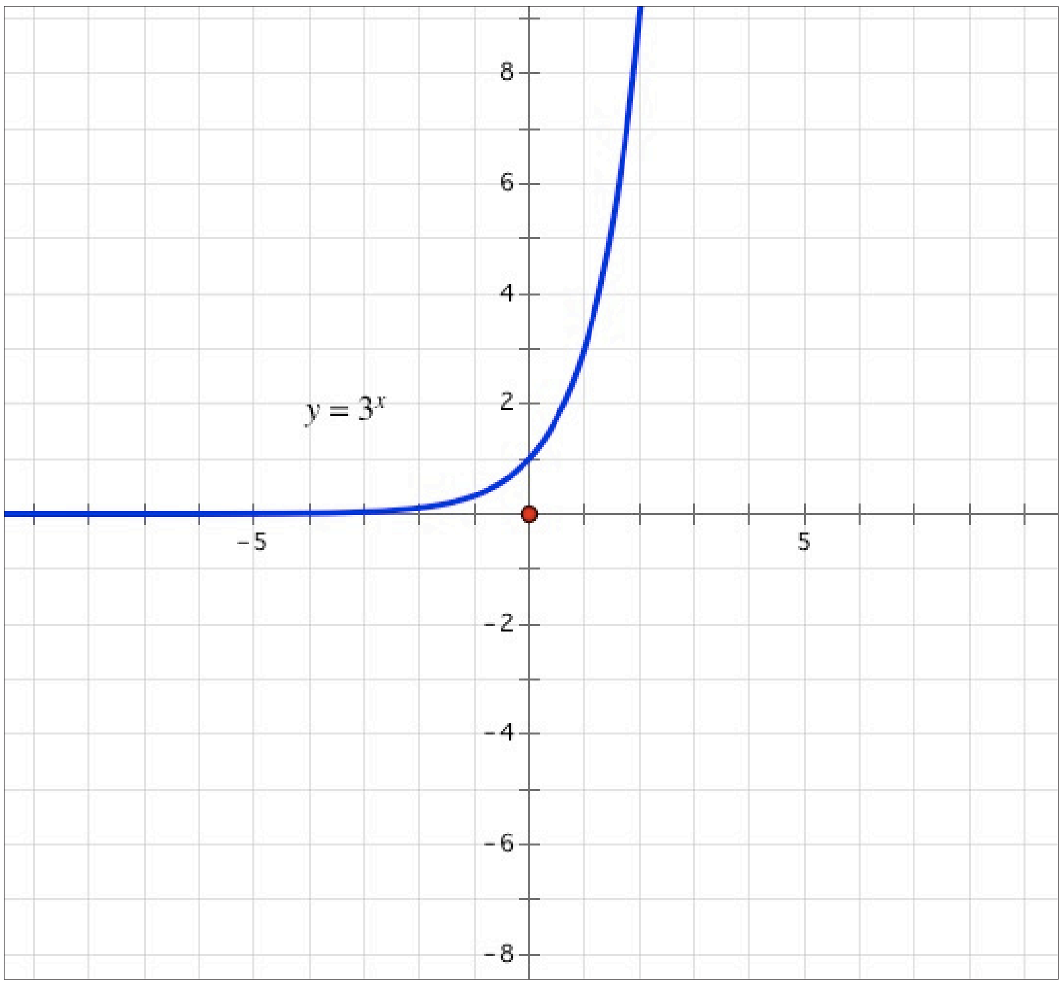
In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #3, will work on altering the quadratic equation whose graph is the basic exponential function. Use separate sheets of graph paper for your work.

Step 1

Each member of the group should sketch a copy of the function $y = 3^x$. (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

Solutions:



Step 2

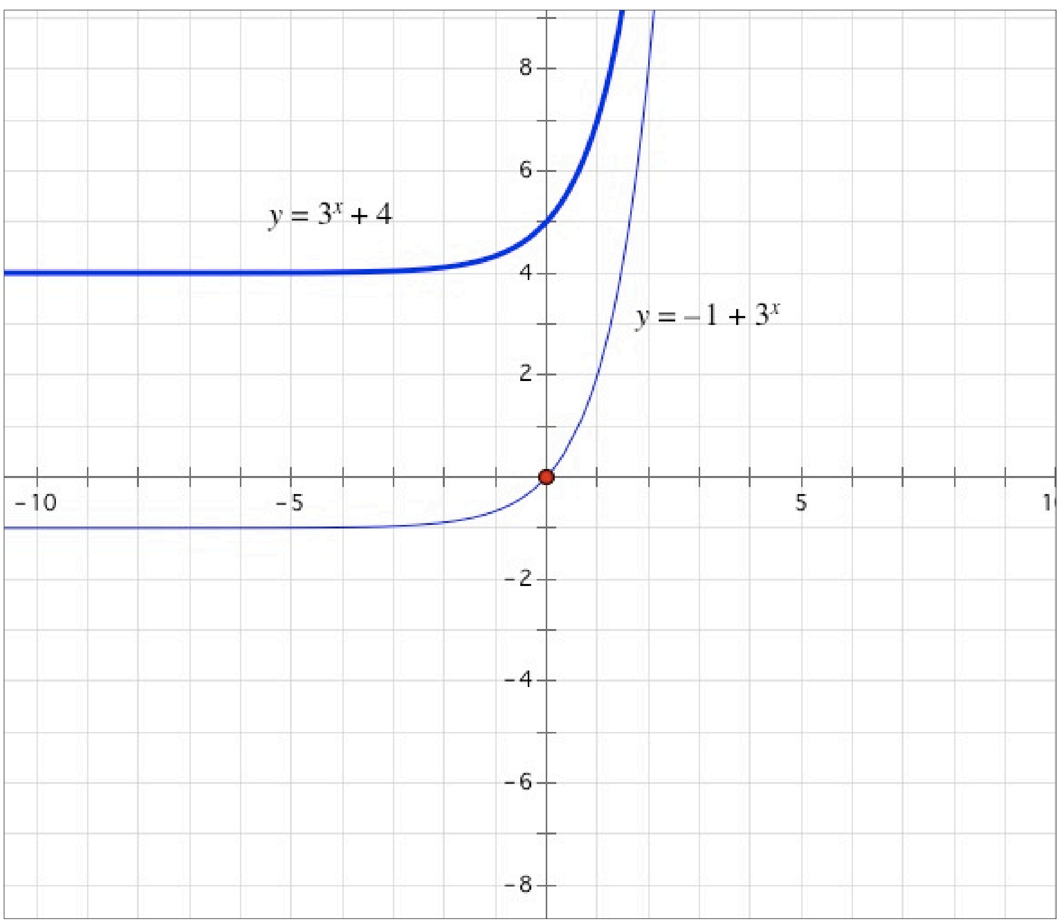
The following problems make four different kinds of changes to the original parent equation of the exponential function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

Problem 1

Graph the following equations on the same axes as the parent function $y = 3^x$:

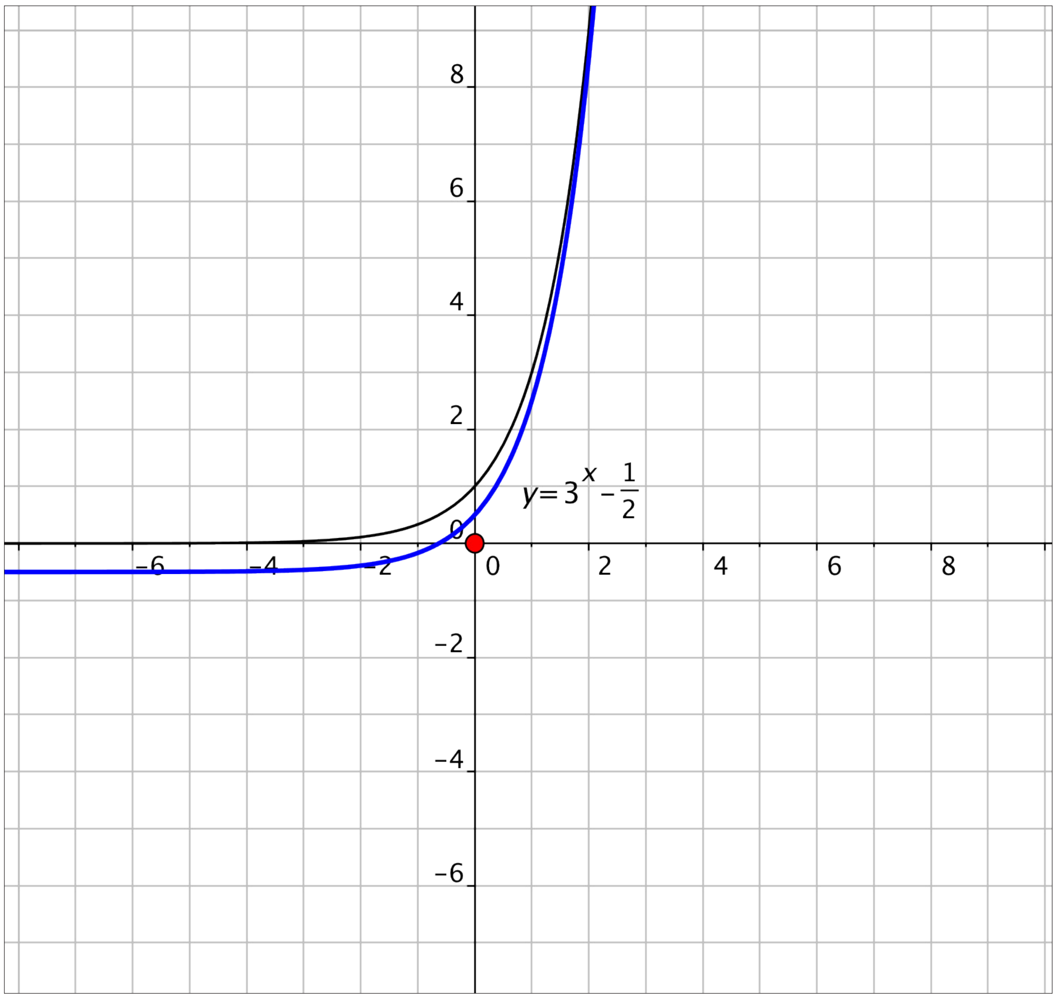
$y = 3^x + 4$ $\qquad$ $y = -1 + 3^x$

Solutions:



Generalize your observations by predicting what the graph of $y = 3^x + c$ will look like in comparison to the parent function $y = 3^x$, no matter what value c has. Test your idea by predicting what the graph of $y = 3^x - \frac{1}{2}$ will look like, and then graph it.

Solutions:

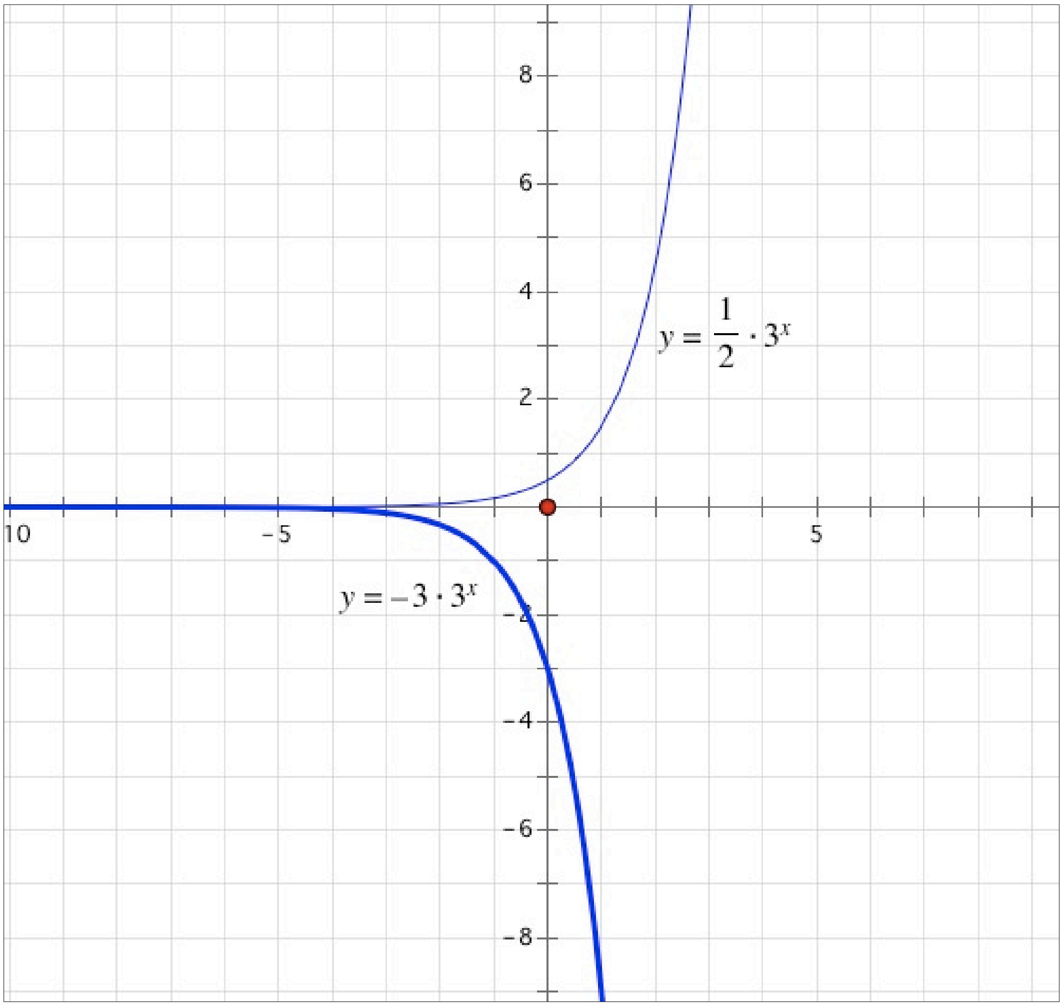


Problem 2

Graph the following equations on the same axes as the parent function $y = 3^x$:

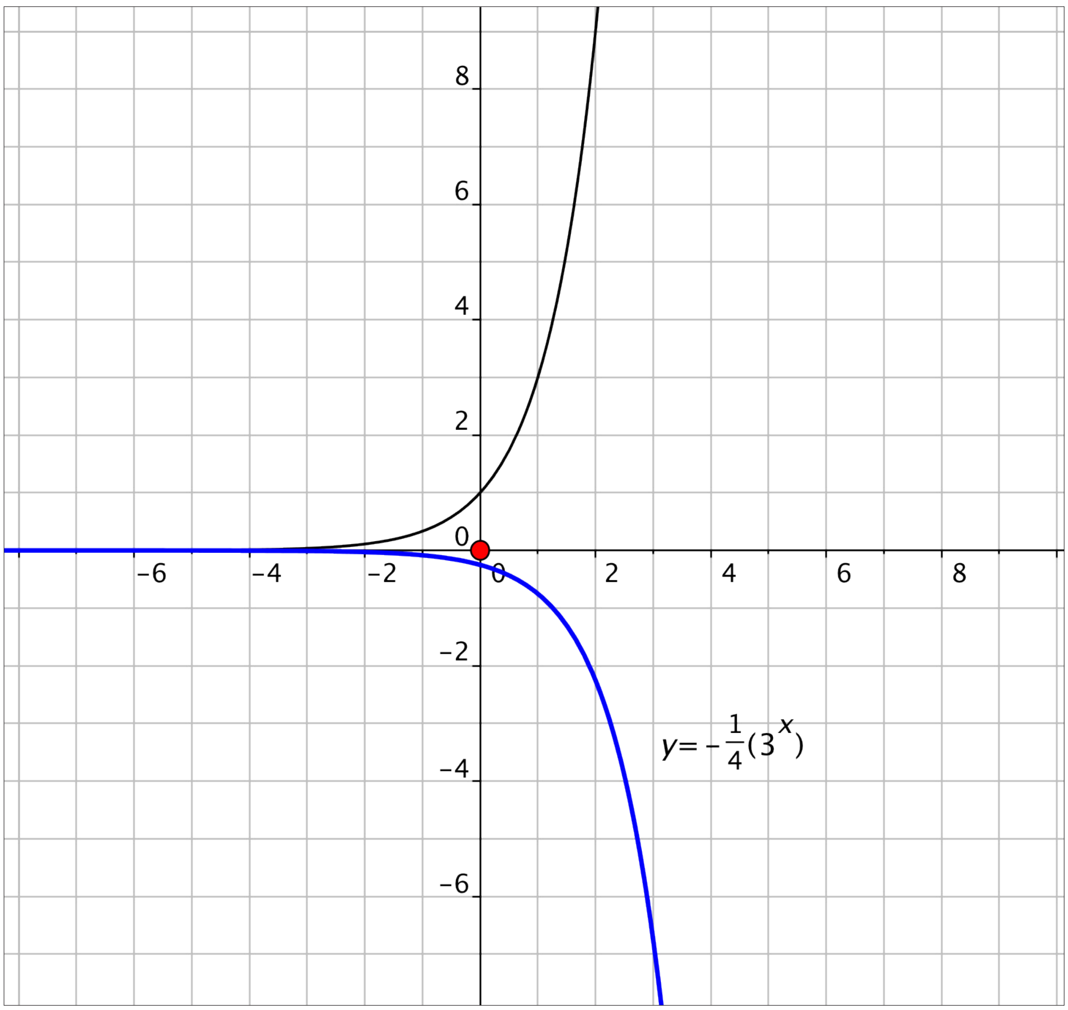
$y = -3(3^x)$ $y = \frac{1}{2}(3^x)$

Solutions:



Generalize your observations by predicting what the graph of $y = c3^x$ will look like in comparison to the parent function $y = 3^x$, no matter what value c has. Test your idea by predicting what the graph of $y = -\frac{1}{4}(3^x)$ will look like, and then graph it.

Solutions:

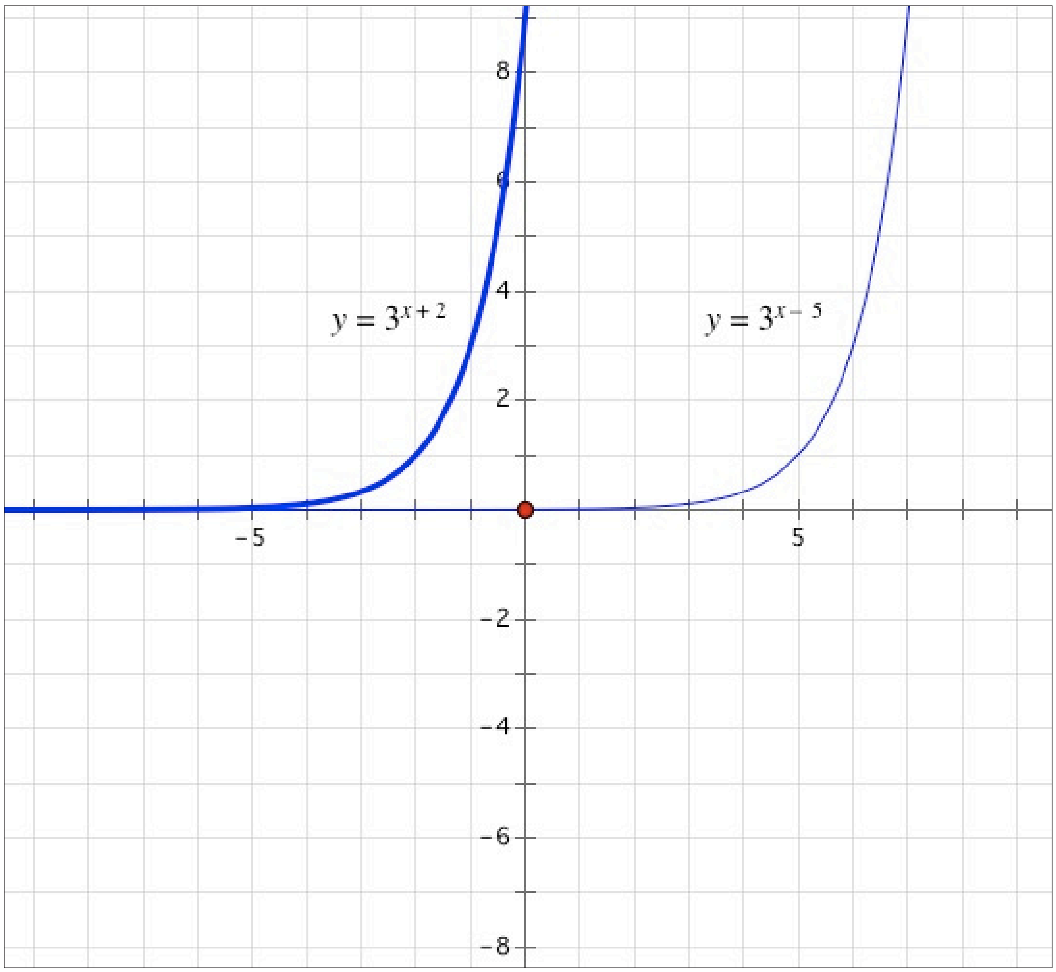


Problem 3

Graph the following equations on the same axes as the parent function $y = 3^x$:

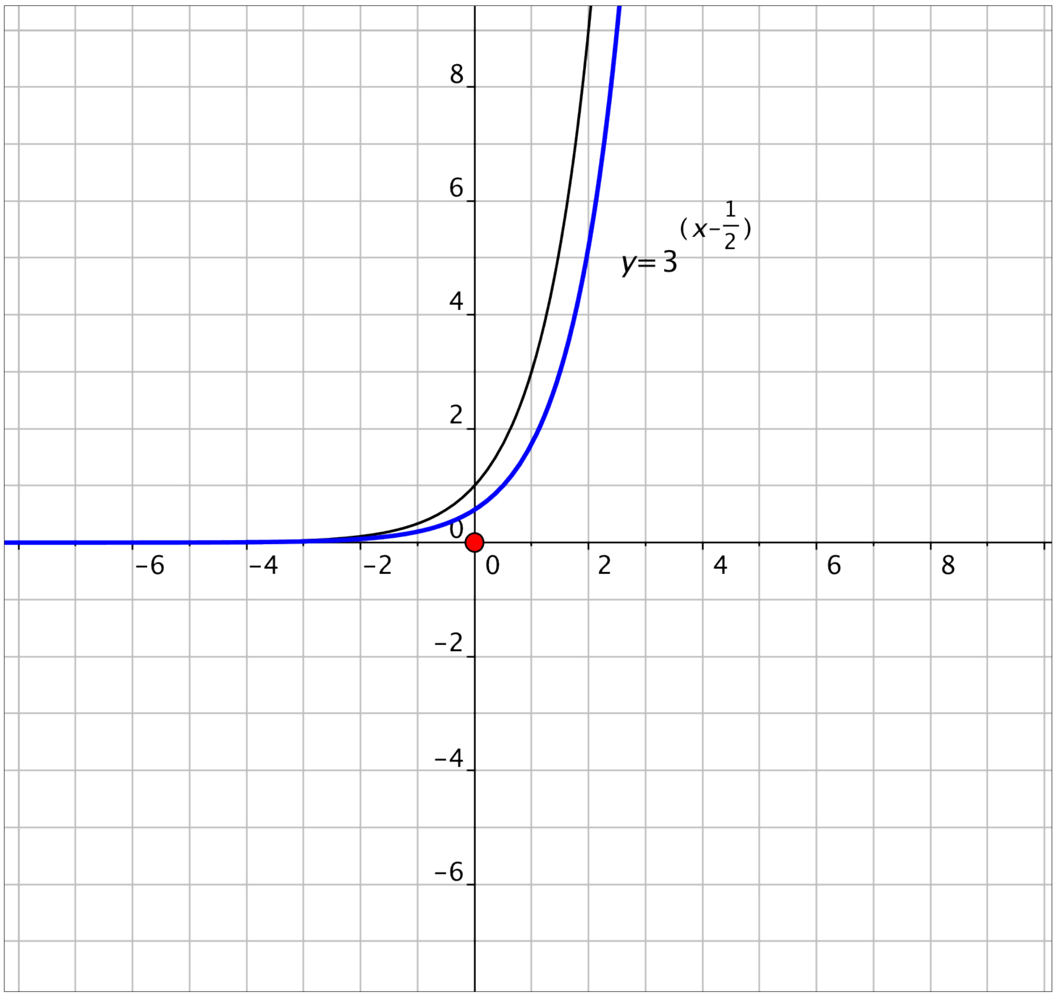
$y = 3^{(x + 2)}$ $y = 3^{(x - 5)}$

Solutions:



Generalize your observations by predicting what the graph of $y = 3^{(x-c)}$ will look like in comparison to the parent function $y = 3^x$, no matter what value *c* has. Test your idea by predicting what the graph of $y = 3^{(x-½)}$ will look like, and then graph it.

Solutions:

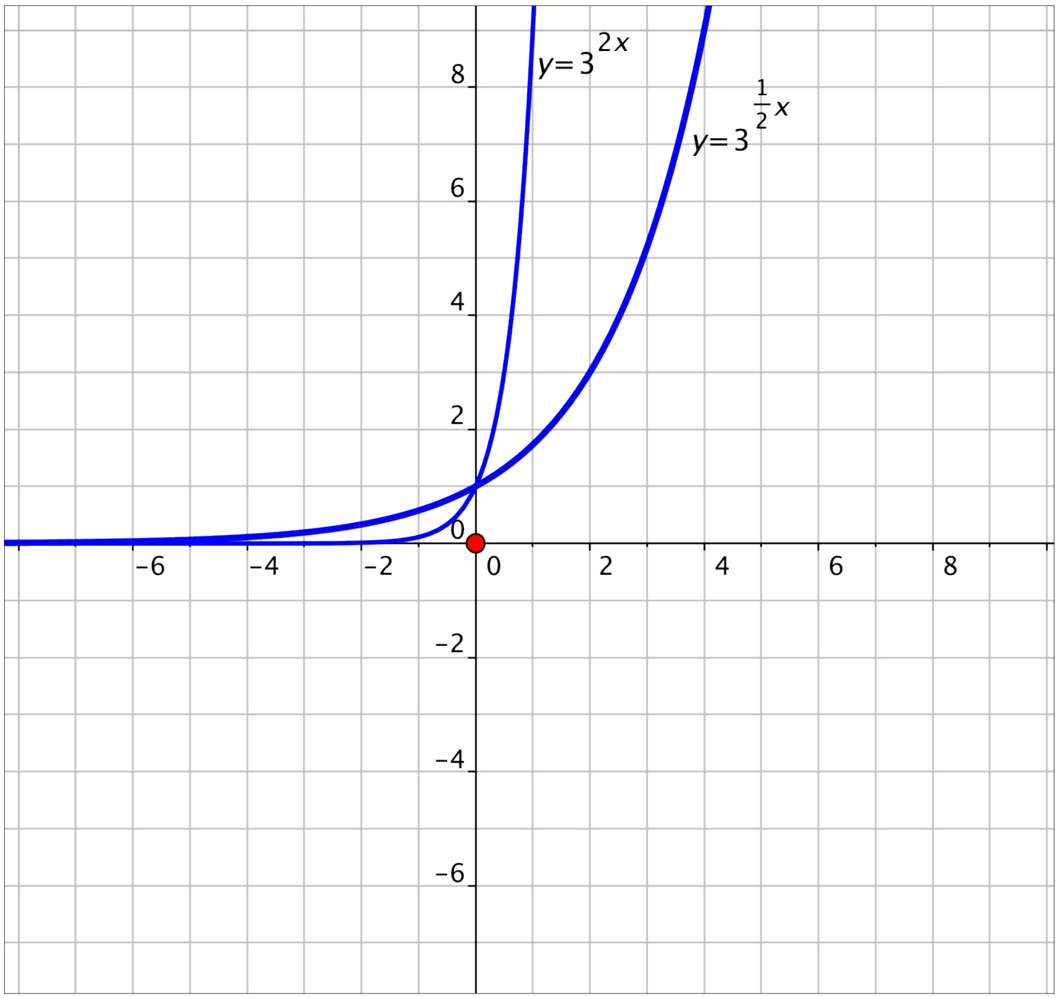


Problem 4

Graph the following equations on the same axes as the parent function $y = 3^x$:

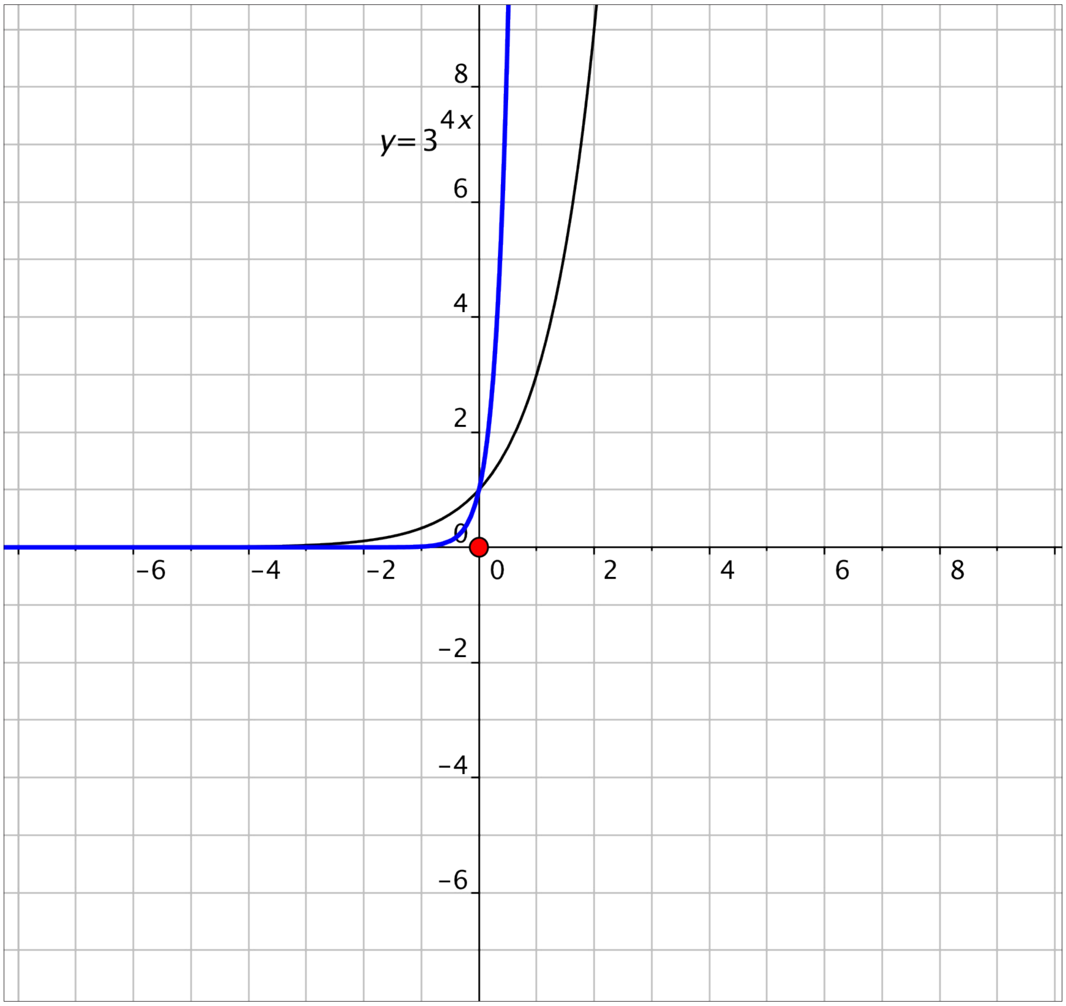
$y = 3^{2x}$ $y = 3^{\frac{1}{2}x}$

Solutions:



Generalize your observations by predicting what the graph of $y = 3^{cx}$ will look like in comparison to the parent function $y = 3^x$, no matter what value c has. Test your idea by predicting what the graph of $y = 3^{4x}$ will look like, and then graph it.

Solutions:



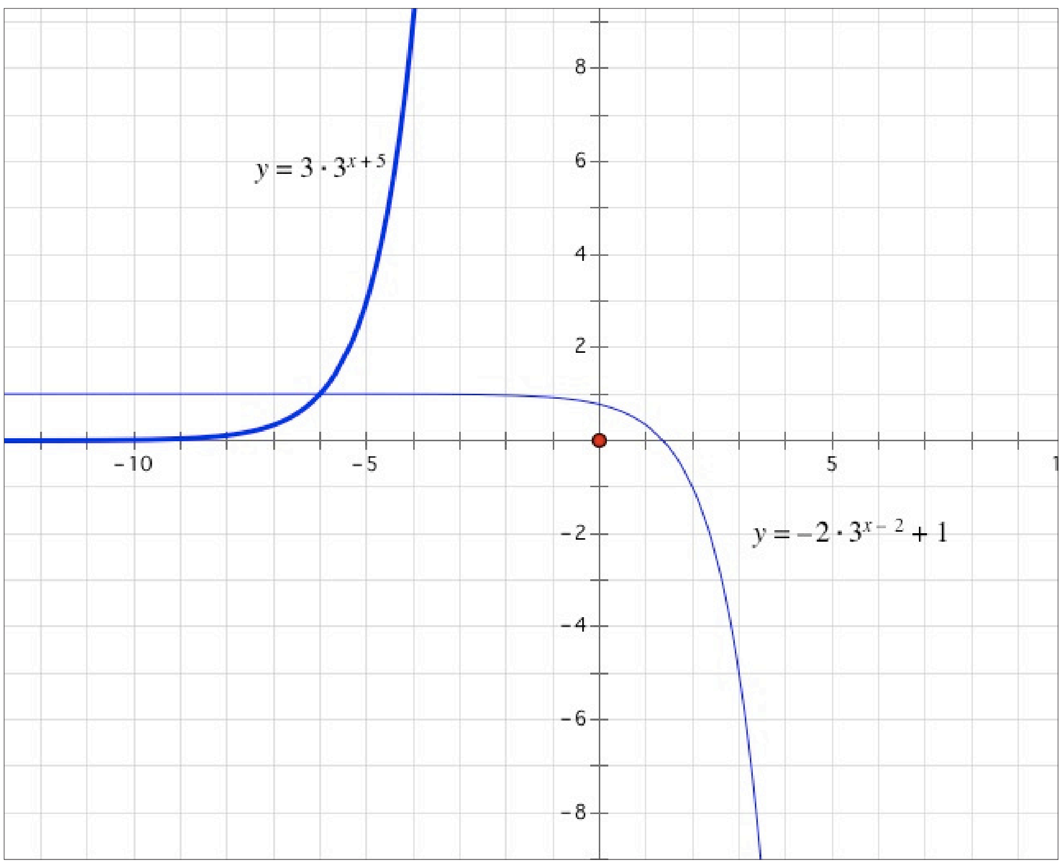
Step 3

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function $y = 3^x$:

$y = 3 \cdot 3^{(x + 5)}$ $y = -2 \cdot 3^{(x - 2)} + 1$

After you've recorded your predictions, test them by graphing each equation.

Solutions:



Handout 5D:
Function Transformations Group 4

Parent function: $y = \left(\frac{1}{3}\right)^x$

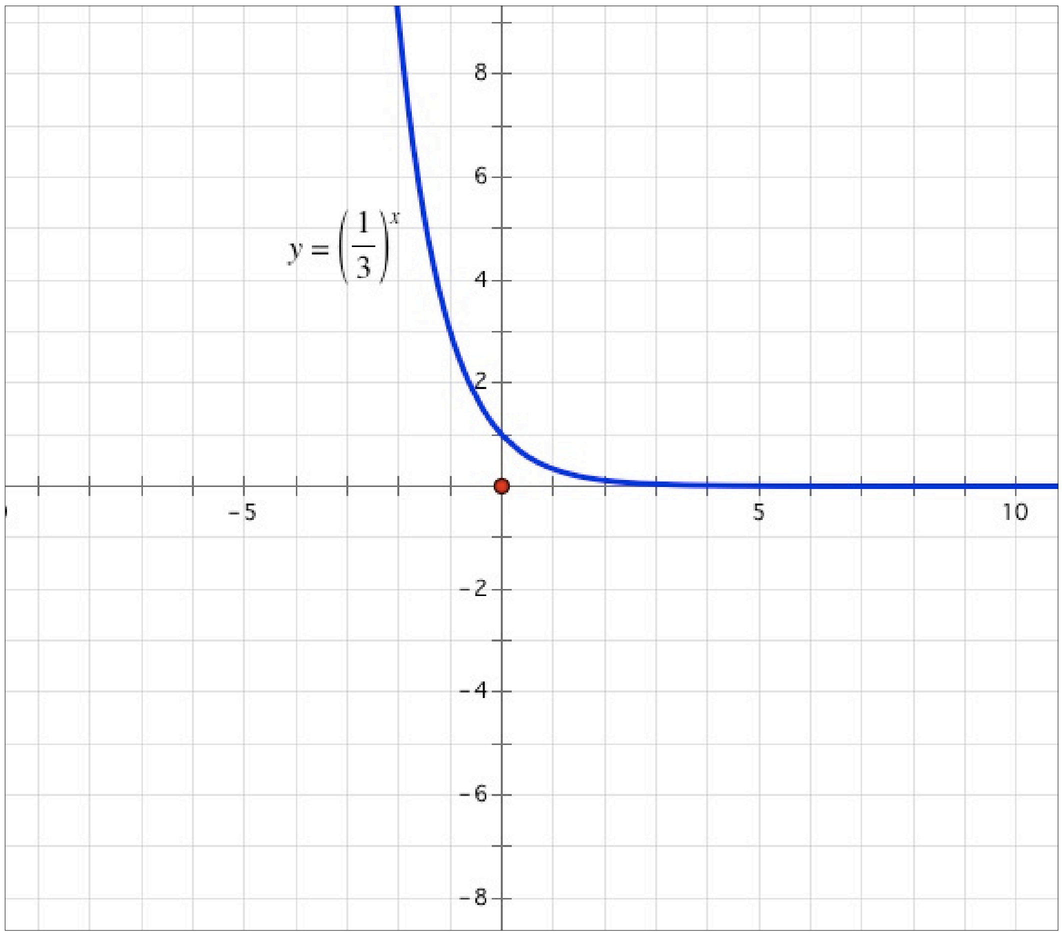
In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #4, will work on altering an equation whose graph is an exponential function. Use separate sheets of graph paper for your work.

Step 1

Each group member should sketch a copy of the function $y = \left(\frac{1}{3}\right)^x$. (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

Solution:



Step 2

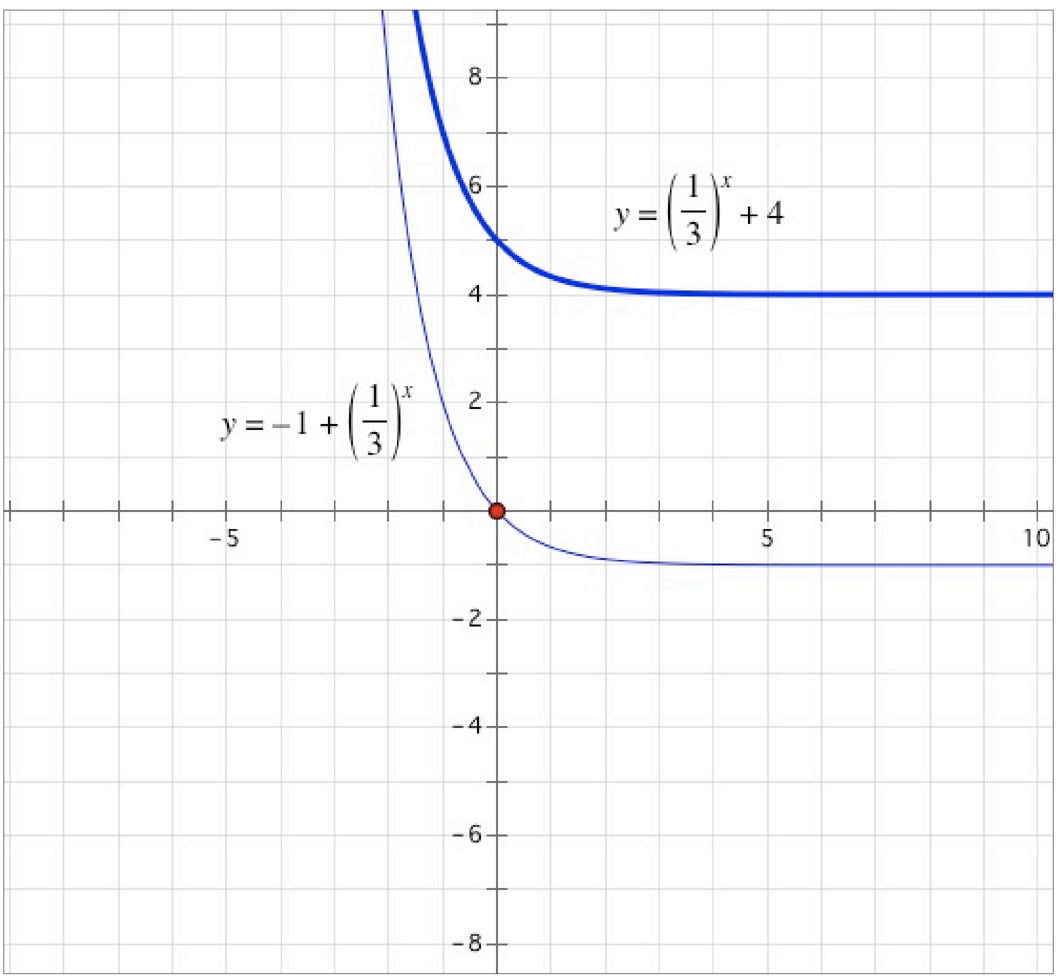
The following problems make four different kinds of changes to the original parent equation of the exponential function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

Problem 1

Graph the following equations on the same axes as the parent function $y = \left(\frac{1}{3}\right)^x$:

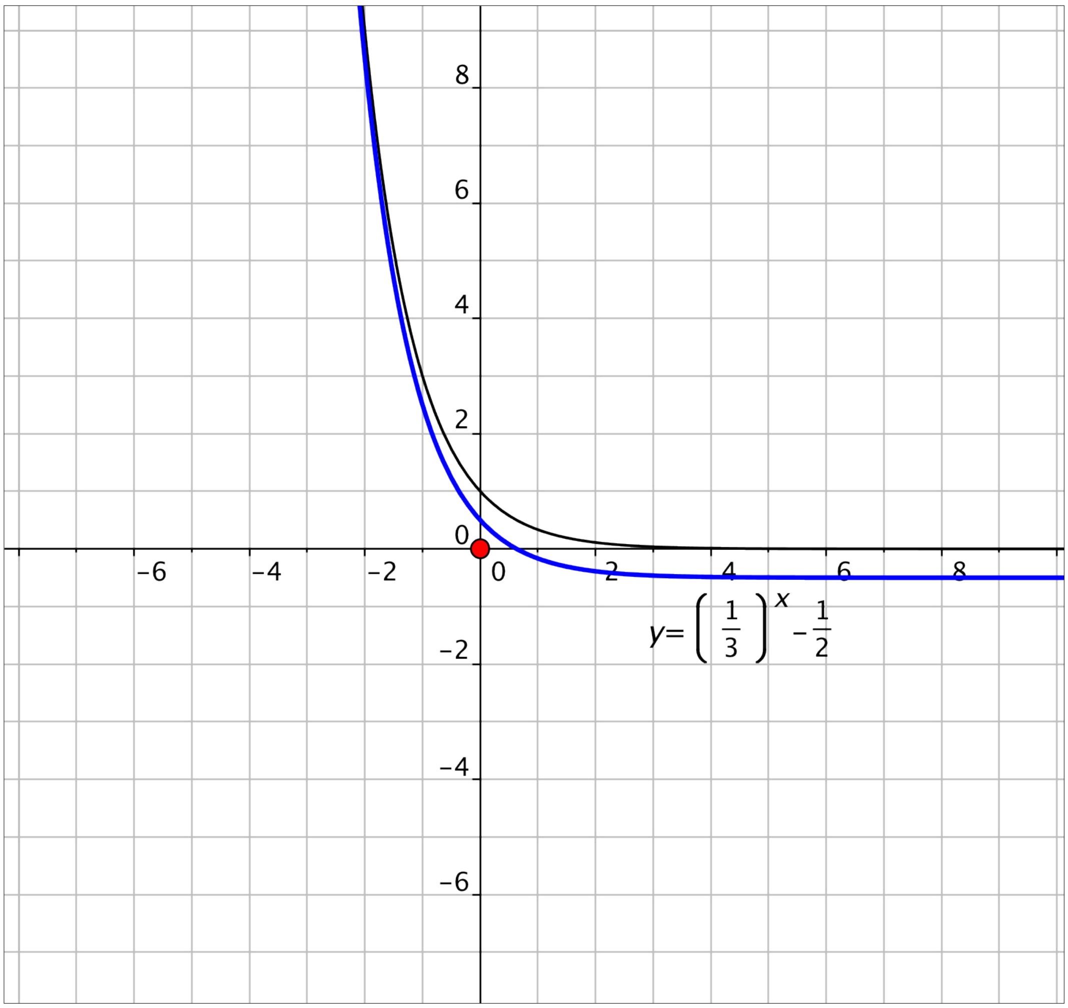
$y = \left(\frac{1}{3}\right)^x + 4$ $\qquad$ $y = -1 + \left(\frac{1}{3}\right)^x$

Solutions:



Generalize your observations by predicting what the graph of $y = \left(\frac{1}{3}\right)x^2 + c$ will look like in comparison to the parent function $y = \left(\frac{1}{3}\right)^x$, no matter what value c has. Test your idea by predicting what the graph of $y = \left(\frac{1}{3}\right)^x - \frac{1}{2}$ will look like, and then graph it.

Solutions:

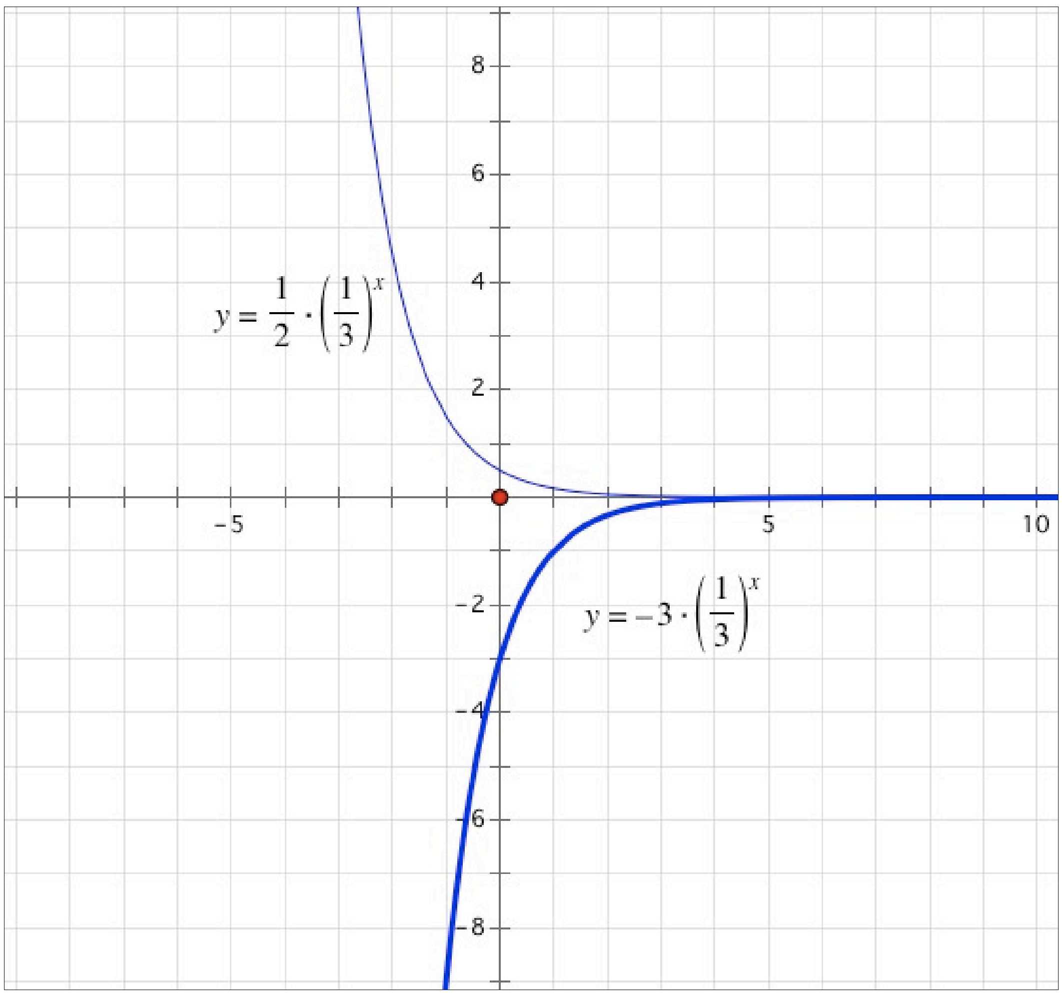


Problem 2

Graph the following equations on the same axes as the parent function $y = \left(\frac{1}{3}\right)^x$:

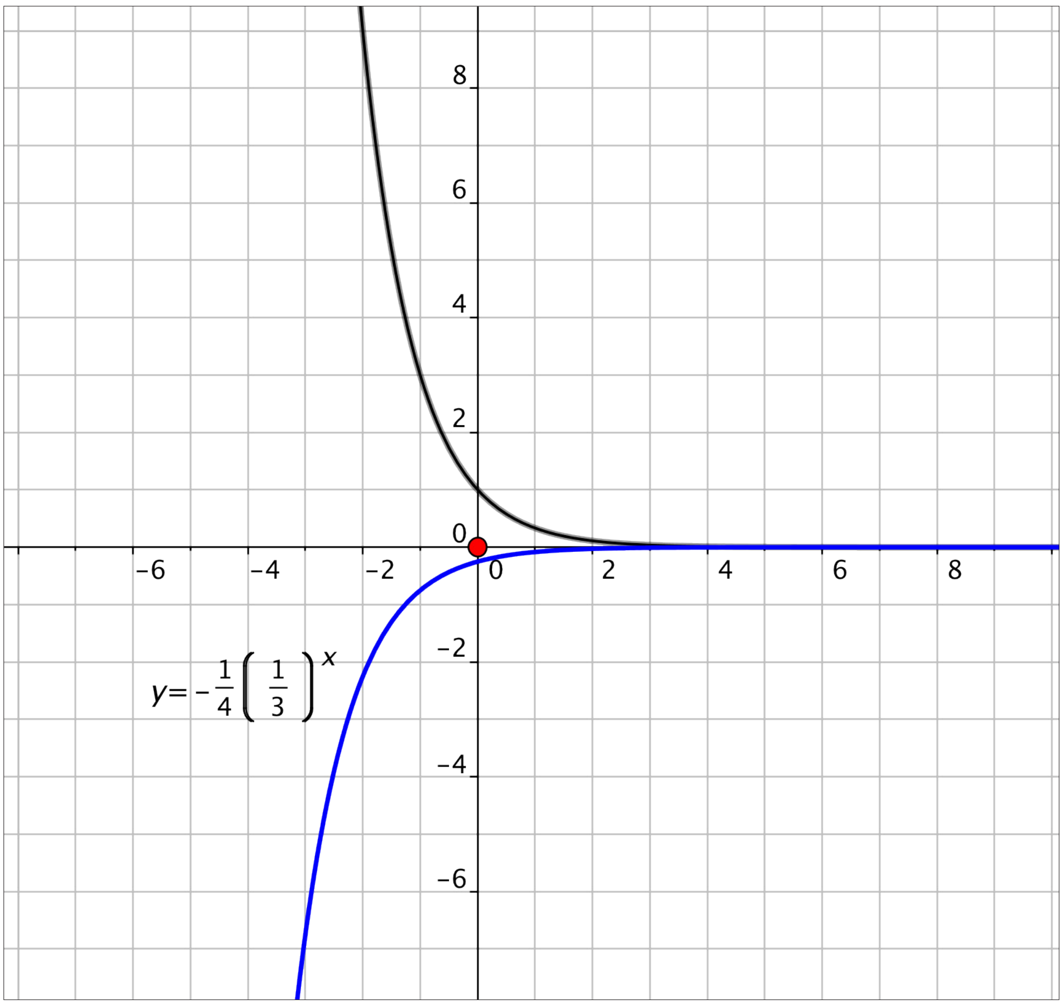
$y = -3\left(\frac{1}{3}\right)^x$ $y = \frac{1}{2}\left(\frac{1}{3}\right)^x$

Solutions:



Generalize your observations by predicting what the graph of $y = c\left(\frac{1}{3}\right)^x$ will look like in comparison to the parent function $y = \left(\frac{1}{3}\right)^x$, no matter what value c has. Test your idea by predicting what the graph of $y = -\frac{1}{4}\left(\frac{1}{3}\right)^x$ will look like, and then graph it.

Solutions:

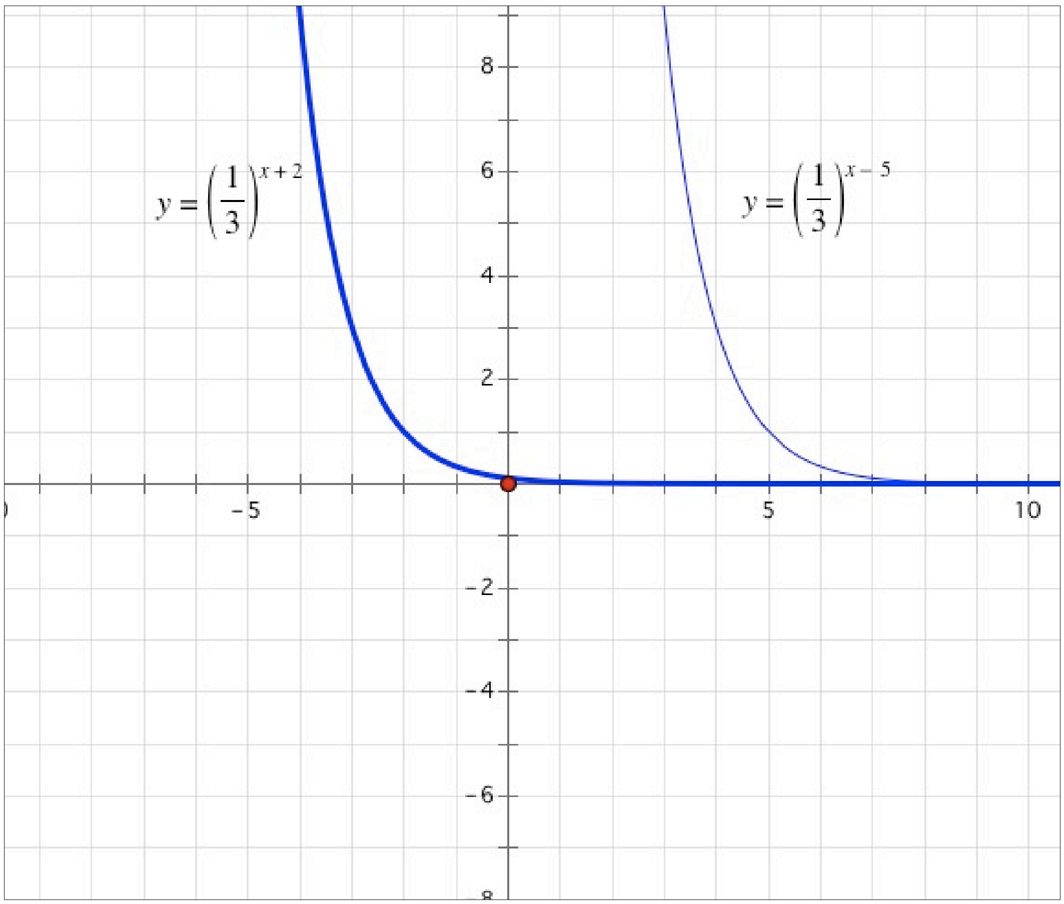


Problem 3

Graph the following equations on the same axes as the parent function $y = \left(\frac{1}{3}\right)^x$:

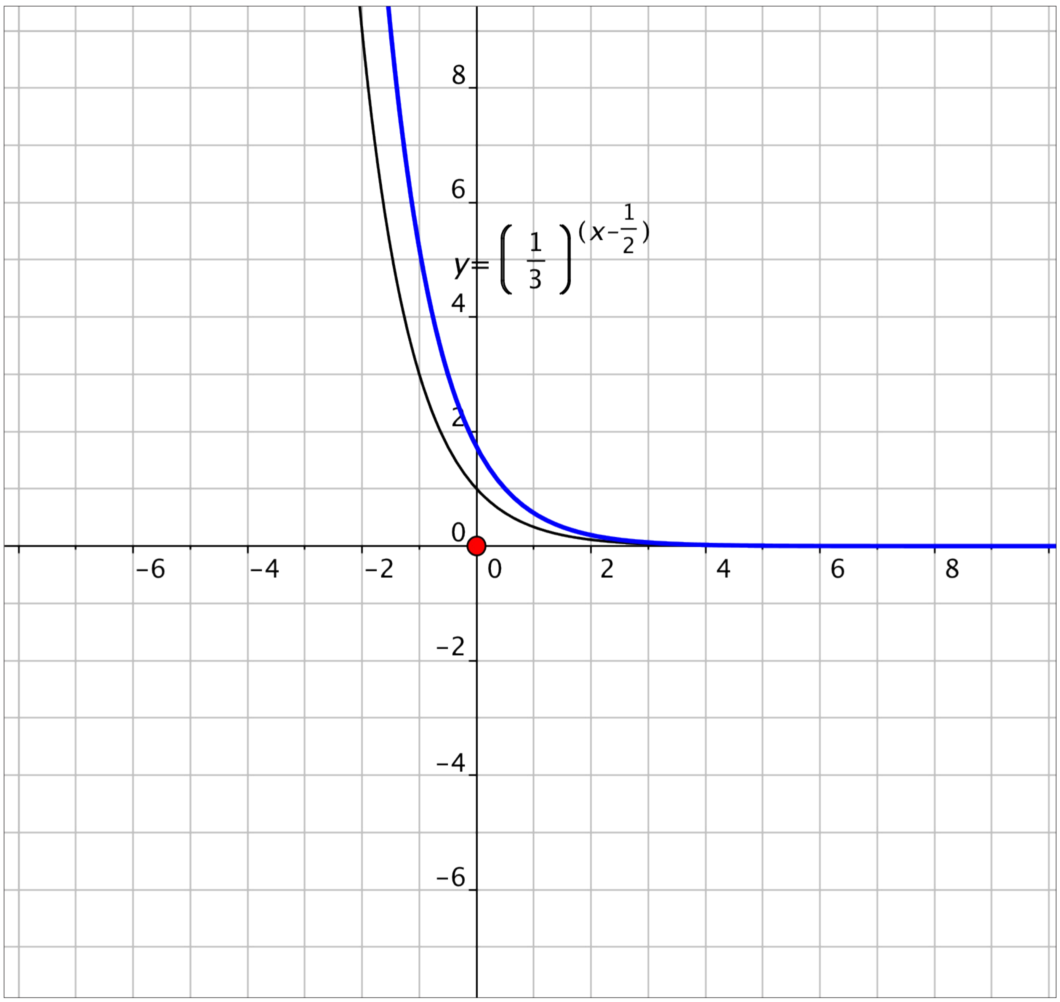
$y = \left(\frac{1}{3}\right)^{(x+2)}$ $y = \left(\frac{1}{3}\right)^{(x-5)}$

Solutions:



Generalize your observations by predicting what the graph of $y = \left(\frac{1}{3}\right)^{(x-c)}$ will look like in comparison to the parent function $y = \left(\frac{1}{3}\right)^{x}$, no matter what value c has. Test your idea by predicting what the graph of $y = \left(\frac{1}{3}\right)^{(x-½)}$ will look like, and then graph it.

Solutions:

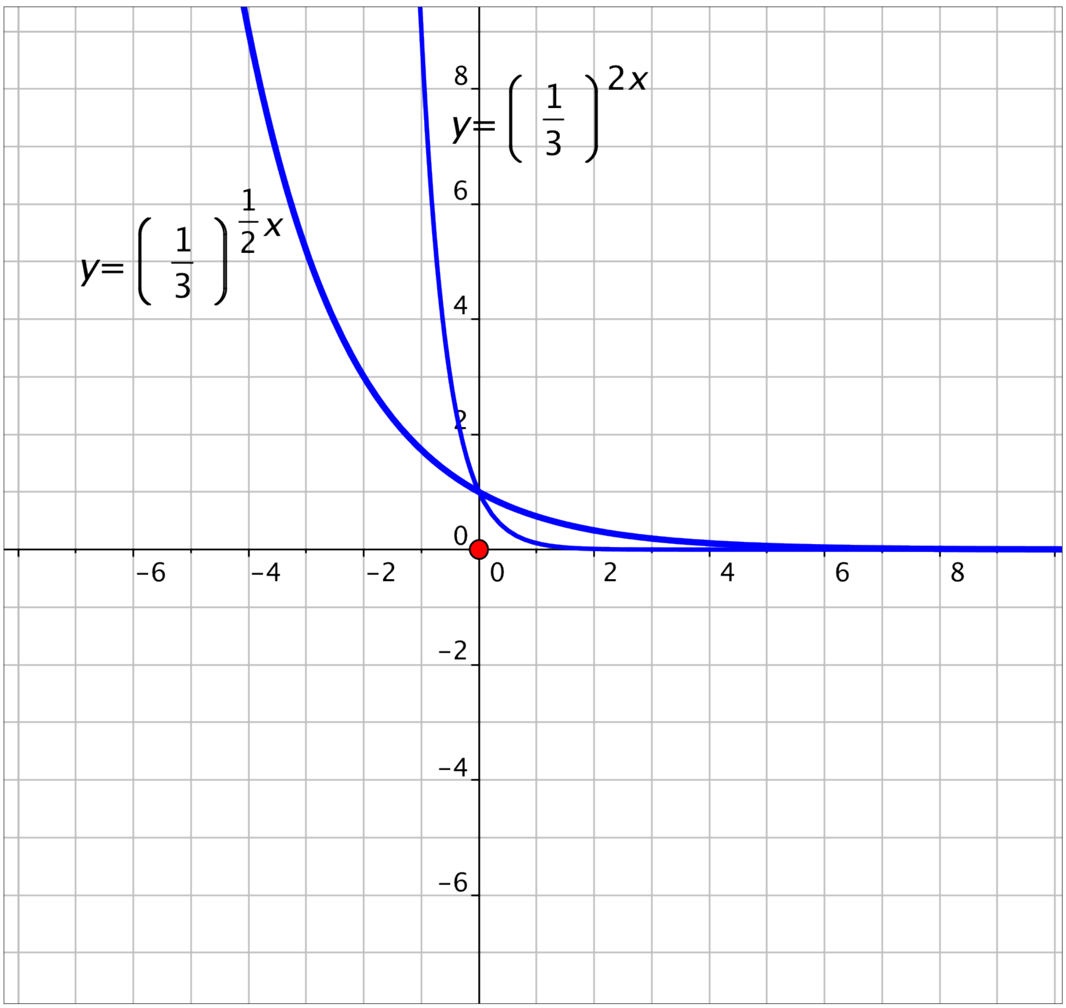


Problem 4

Graph the following equations on the same axes as the parent function $y = \left(\frac{1}{3}\right)^x$:

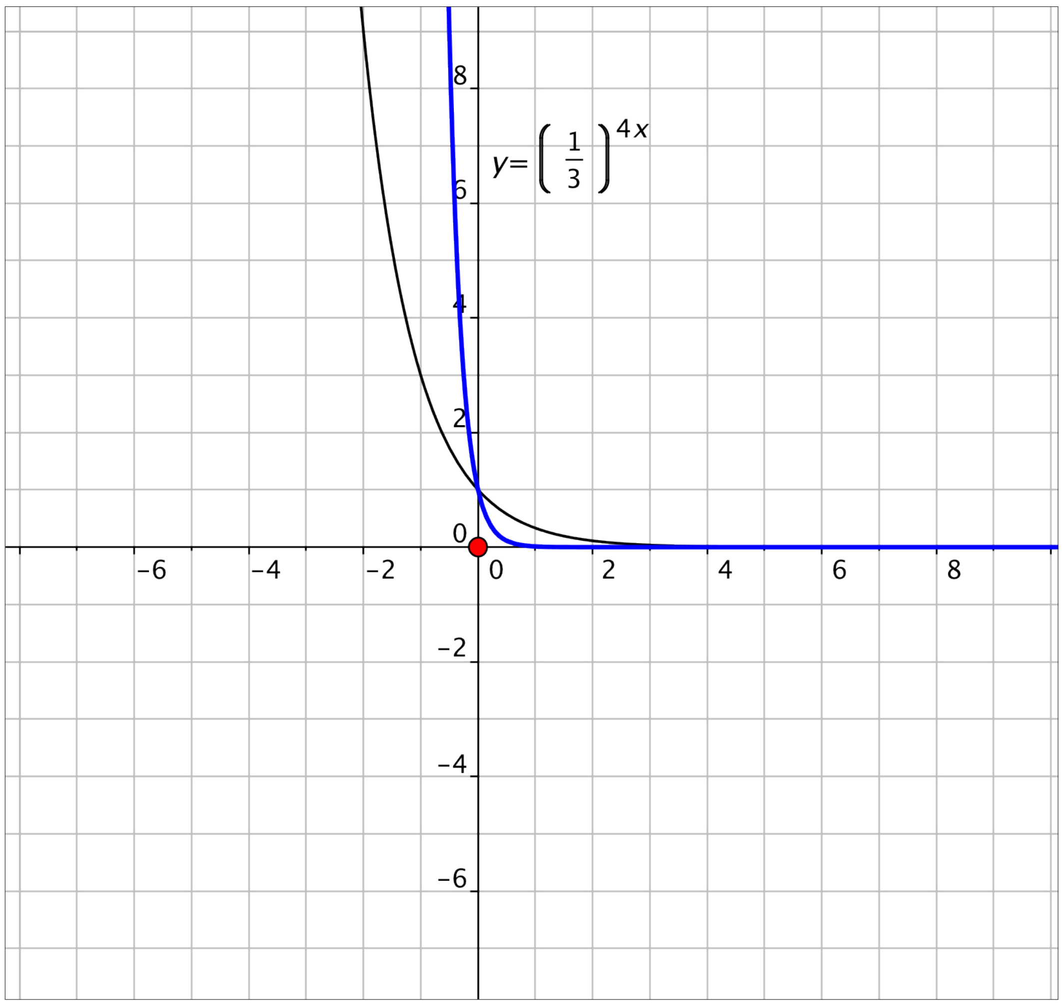
$y = \left(\frac{1}{3}\right)^{2x}$ $y = \left(\frac{1}{3}\right)^{½x}$

Solutions:



Generalize your observations by predicting what the graph of $y = \left(\frac{1}{3}\right)^{cx}$ will look like in comparison to the parent function $y = \left(\frac{1}{3}\right)^{x}$, no matter what value c has. Test your idea by predicting what the graph of $y = \left(\frac{1}{3}\right)^{4x}$ will look like, and then graph it.

Solutions:



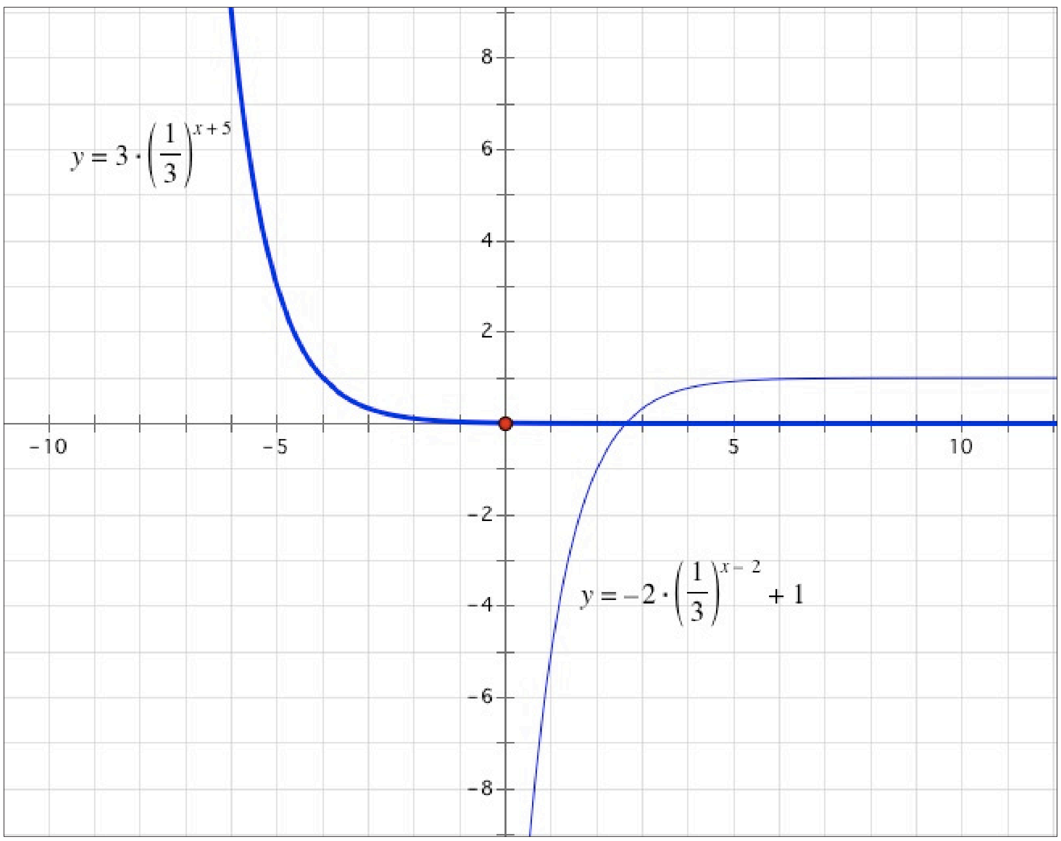
Step 3

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function $y = \left(\frac{1}{3}\right)^x$:

$y = 3\left(\frac{1}{3}\right)^{(x+5)}$ $y = -2\left(\frac{1}{3}\right)^{(x-2)} + 1$

After you've recorded your predictions, test them by graphing each equation.

Solutions:



Activity 2B: Changing Sine

Students apply the same transformations they used in Activity 2A to their graph of the sine function. Using graphing calculators, students observe how changing coefficients in the equation $y = \sin(x)$ affects the graph.

Understandings

- The sine function can be transformed by adding a constant (a vertical shift), multiplying by a constant (a vertical shift), subtracting a constant from the independent variable (a horizontal shift), and multiplying the independent variable by a constant (a horizontal stretch).

Materials Needed

- **Handout 6: Changing Sine**
- Graphing calculators
- Graph paper
- Students' copies of **Handout 5A–D: Function Transformations Group 1–4**
- Students' copies of **Handout 1: Exploring Sound**

1. Introduce the activity and distribute materials.
Tell students that they will transform the graph of a sine wave, much like they did with different functions in Activity 2A. Students will use graphing calculators to generate transformed sine functions, copy the resulting graphs onto graph paper, and summarize the effects of various coefficients in a sine function on the graphs of the parent function $y = \sin(x)$. Give each student **Handout 6: Changing Sine**, a graphing calculator, and some graph paper.

> **Note:** At this point, students have not yet seen the general form of the sine function. You will introduce this after students transform the sine wave.

2. Work as a whole class on Part 1 of Handout 6.
Work through Part 1 with students, and troubleshoot any calculator problems that arise.

> **Note:** The equations provided assume that students are using the radian mode in their graphing calculators. Students will need to refer to their past work from Activities 1A and 2A.

3. Have students complete Handout 6.
Although they may work individually, encourage students to share their findings as they explore the effects of different coefficients on the graphs of the parent function $y = \sin(x)$.

4. Introduce the general form of the sine function.
Write the general form of the sine function on the board and ask students to share the effects they noticed when changing the value of *A*, *B*, *C*, or *D*. Make parallels when possible between the effects of these values and the form of the transformed equations from Activity 2A. Note the specialized language that is used for trigonometric functions—for example, the term *phase shift* is often used instead of *horizontal shift*, a vertical stretch changes the *amplitude*, and a horizontal stretch changes the *period* of the function.

Teacher's Notes: Frequency and Period

The frequency of a sine wave describes how frequently a wave occurs. The unit is waves per radian. The *period* is the width of a complete cycle. The frequency (f) and the period (p) have an inverse relationship: $f = 1/p$. For example, if the period is 2π radians, the frequency is 1/(2π) waves per radian.

Sound waves are typically described as having a particular frequency using the unit hertz, which is oscillations/second. When the independent variable for a sine wave is time in seconds, then a radian is a second, so 1 wave per radian is the same as 1 hertz.

You may want to discuss this with students when you go over Part 2 of Handout 6, the effect of B in $y = \sin(Bx)$, because this transformation determines the period (and therefore the frequency) of the wave, using the formula $p = 2\pi/B$.

Handout 6:
Changing Sine

In the following activity, you'll explore transformations of the sine function and the effect of these transformations on both the graph of the function and its accompanying equation. The transformations you apply in this handout are the same transformations you worked with in Handout 5. However, here you will change the parent equation of the sine function to generate equations that can be used to represent sound waves. Look for patterns between the transformations you apply to the sine function and those you have applied to other familiar functions.

As you create the graphs, it will be useful to see at least one cycle of the sine function. To start, your x-axis should go from 0 to 2π (around 6.3) radians. Use a scale from –3 to 3 for the y values. Depending on the equation, you may want to adjust the x-axis scale.

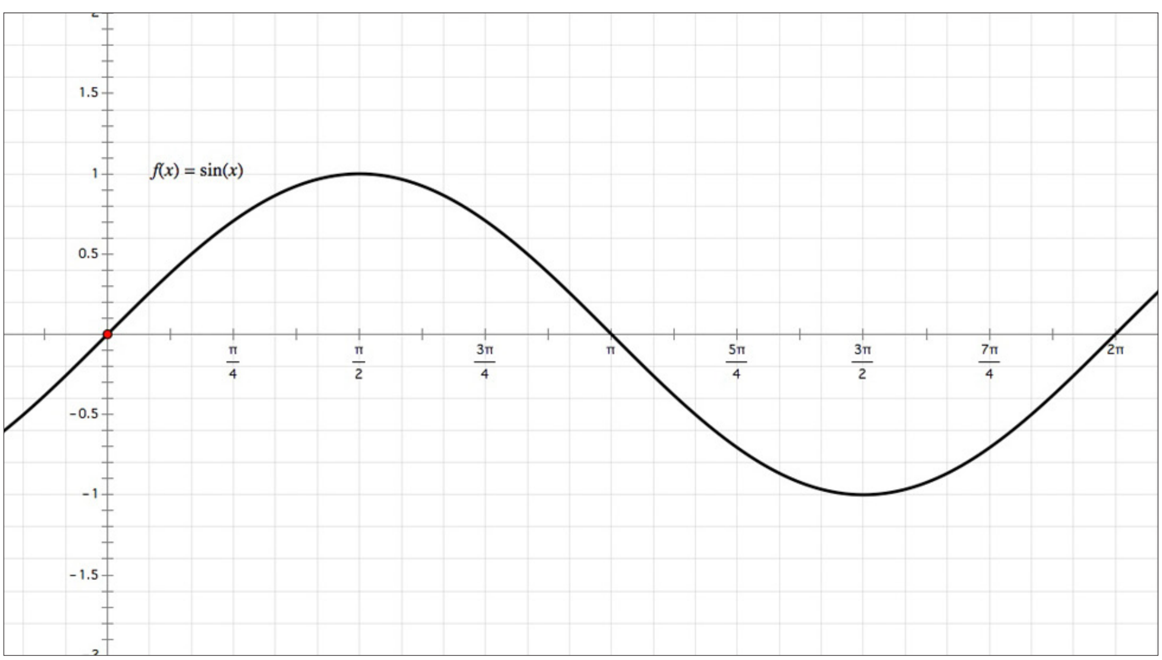
If you are using a graphing utility, set its mode to radians and check the window of the graph. To view different graphs of the equations you generate, you can adjust your window.

Part 1: The effect of A in $y = A\sin(x)$

Stretch the graph of the parent function $y = \sin(x)$ and explore the effect of changing the value of A in the equation $y = A\sin(x)$.

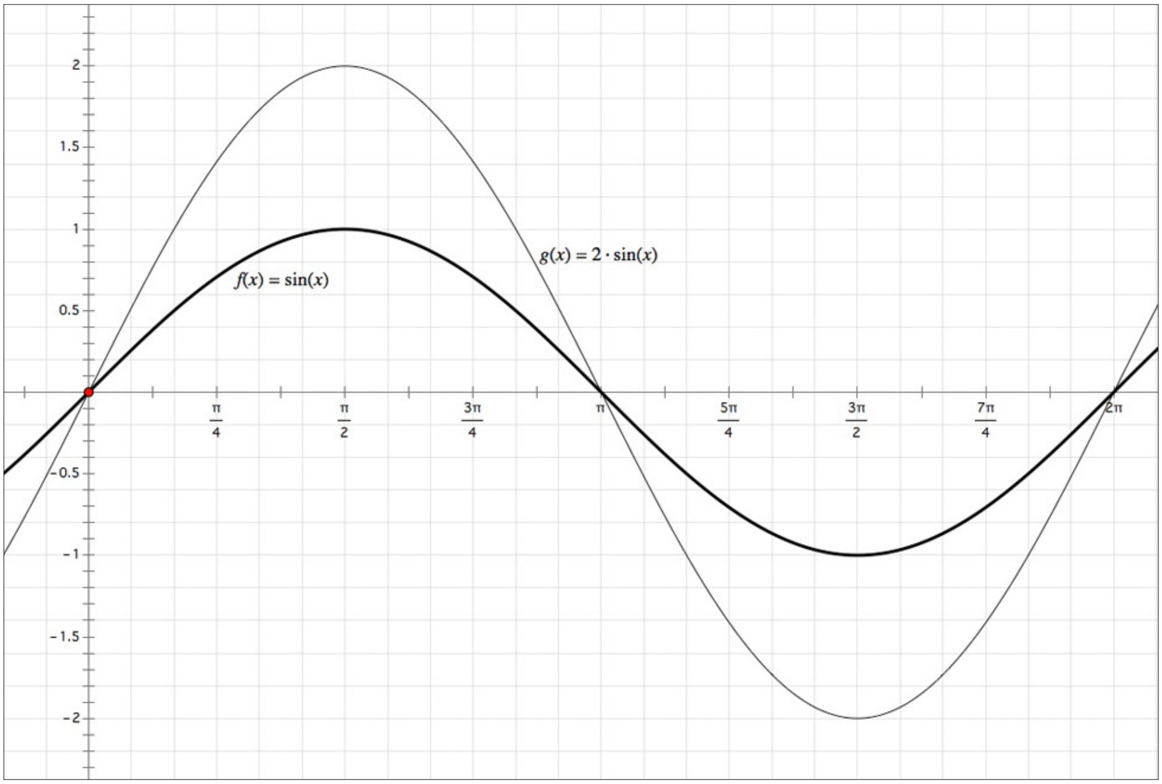
1. Sketch the graph of $y = \sin(x)$ on graph paper.

   Solution:



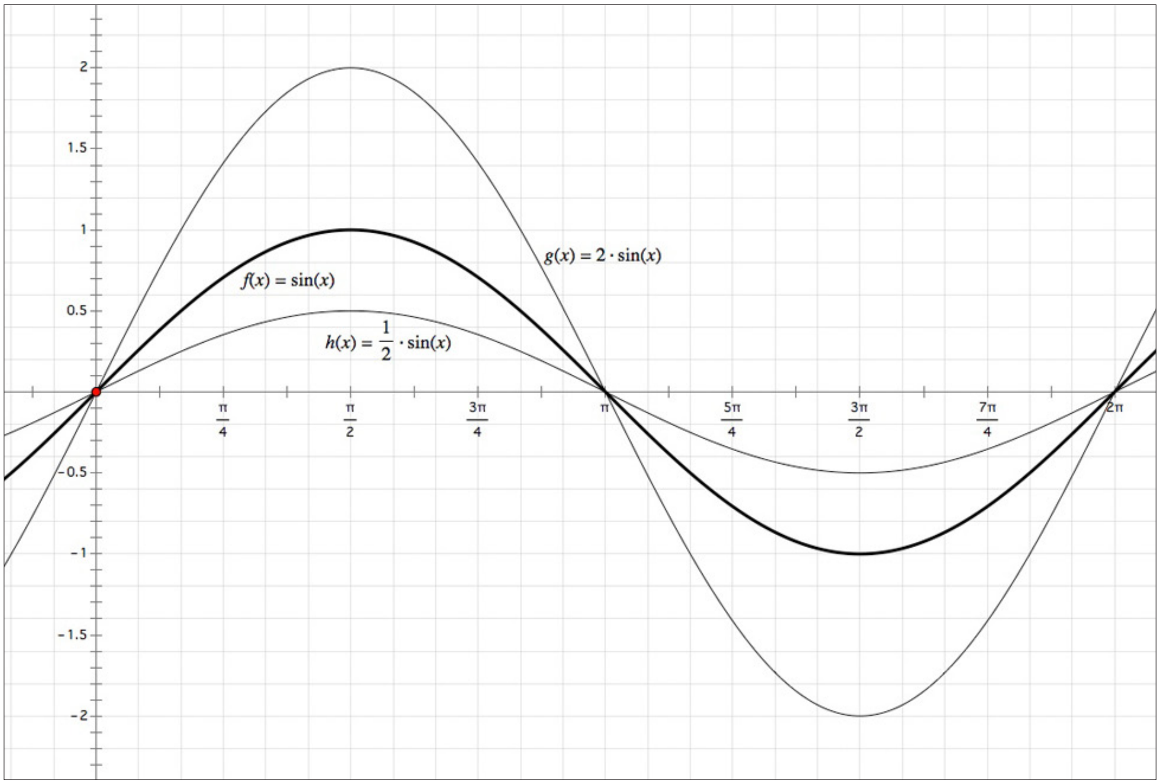
2. Graph $y = 2\sin(x)$ onto the same set of axes as the parent function and label it.

Solution:



3. Graph $y = \frac{1}{2}\sin(x)$ onto the same set of axes and label it. (You'll have three graphs on one set of axes.)

Solution:



4. What can you conclude about the way the coefficient A in the equation above affects the graph of the basic sine function? In general, what effect does multiplying a function by a constant have on the function's graph? Use additional evidence from Handout 5 to back up your conclusion.

Answer: Multiplying sin(x) by a value, A, changes the maximum and minimum y or output values to be A and –A. In general, multiplying a function by a constant stretches the height of the graph if the absolute value of the constant is greater than 1 and compresses it if the absolute value of the constant is less than 1. The graphs in problem 2 on Handout 5 follow this pattern as well.

5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in *A* might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

Answer: A larger height means a louder volume, so if |A| > 1, the sound will be louder than the parent function. If |A| < 1, the sound will be quieter.

Part 2: The effect of *B* in *y* = sin(*Bx*)

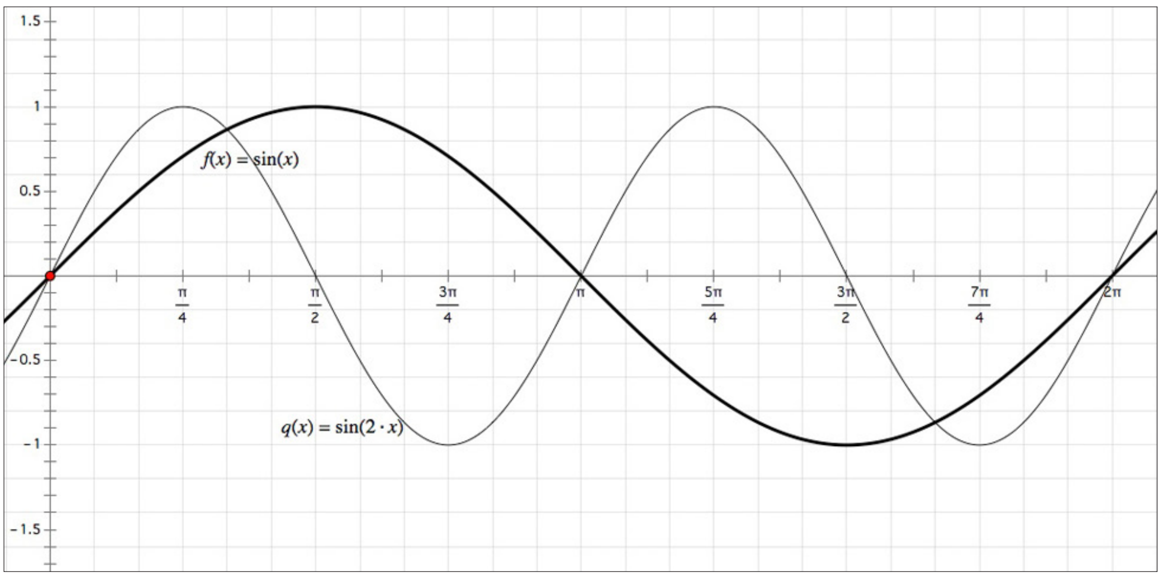
Stretch the graph of the parent function $y = \sin(x)$ and explore the effect of changing the value of B in the equation $y = \sin(Bx)$.

1. Sketch the graph of $y = \sin(x)$ on graph paper.

   Solution: See the graph on page 80.

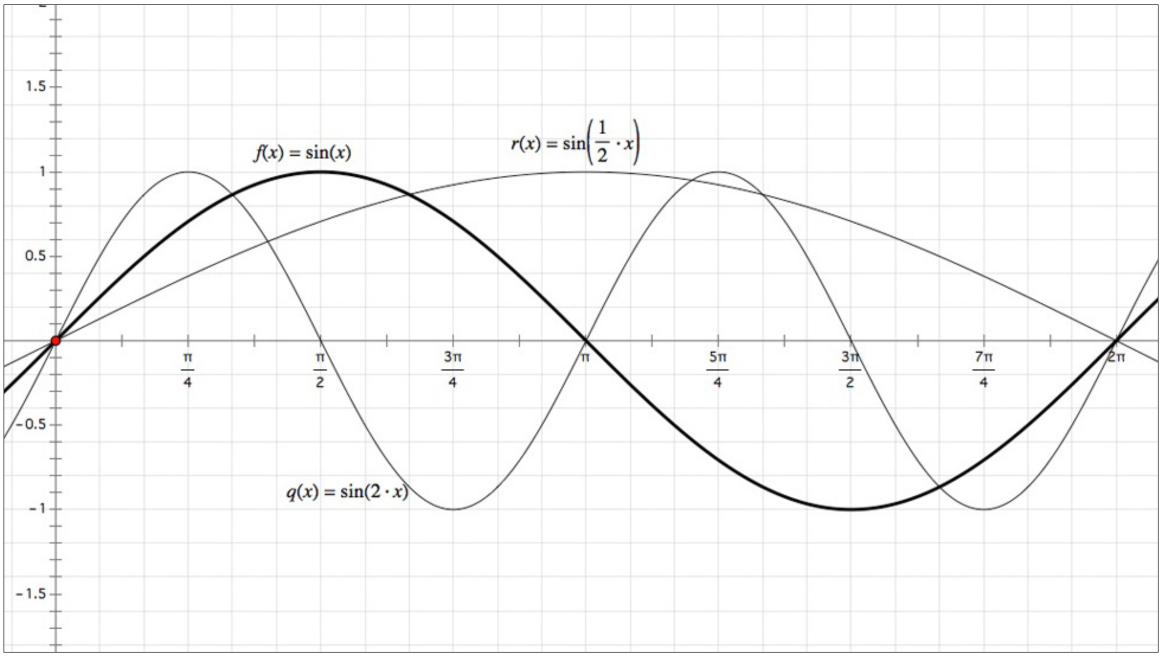
2. Graph $y = \sin(2x)$ onto the same set of axes as the parent function and label it.

   Solution:



3. Graph $y = \sin(\frac{1}{2}x)$ onto the same set of axes and label it. (You'll have three graphs on one set of axes.)

   Solution:



4. What can you conclude about the way the value of *B* in the equation above affects the graph of the basic sine function? In general, what effect does multiplying the independent (input) variable by a positive constant have on the parent function's graph? Use additional evidence from Handout 5 to back up your conclusion.

   Answer: The value changes the period of the function. If |B| > 1, the period is smaller—that is, the cycle repeats more frequently than the parent function. If |B| < 1, the period is larger and the cycle repeats less frequently. In general, multiplying x by a constant stretches or compresses the graph in a horizontal direction. The graphs in problem 4 on Handout 5 follow this pattern as well.

   Note: In this example, students have only worked with positive constants. The sign of the constant can have an effect, though it is not always visible in the graph. You may want to point out that we don't yet know what the effect of a negative constant will be—exploring this could be a good project for students who need an additional challenge. Multiplying the independent variable in a function by a negative constant reflects the graph across the *y* axis, but for even functions like $y = x^2$, $f(-x) = f(x)$ and there is no visible effect, because the graph is reflected onto itself.

5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in *B* might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

   Answer: Changing B will change the pitch of the sound. A greater value of B will give a higher pitch.

Part 3: The effect of *C* in *y* = sin(*x* – *C*)

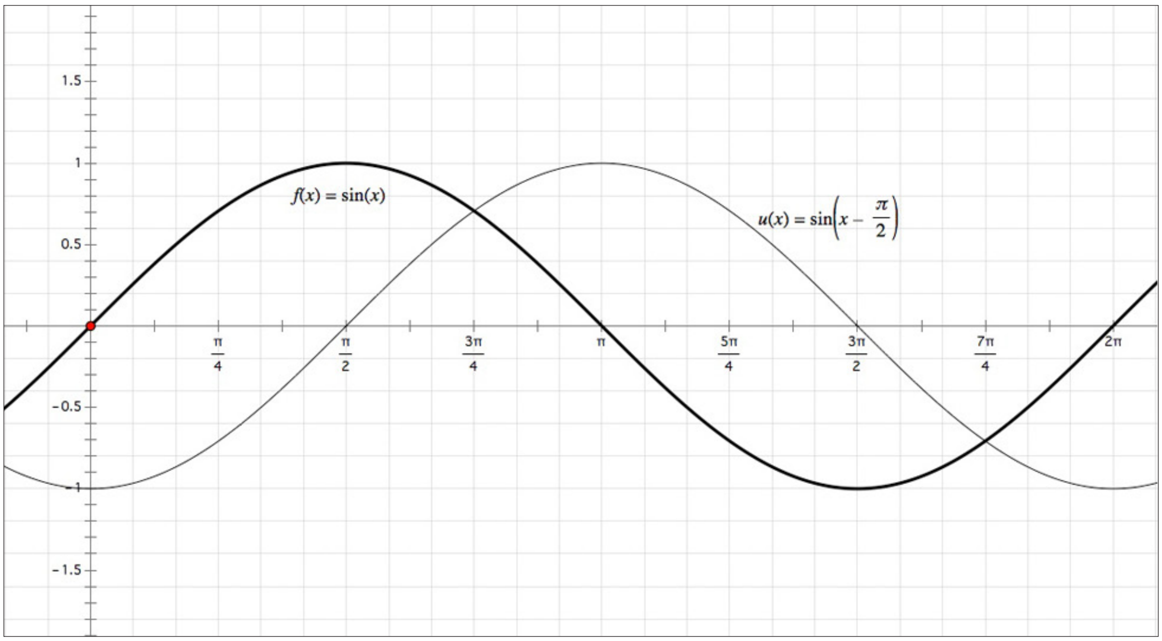
Stretch the graph of the parent function $y = \sin(x)$ and explore the effect of changing the value of C in the equation $y = \sin(x - C)$.

1. Sketch the graph of $y = \sin(x)$ on graph paper.

   Solution: See the graph on page 80.

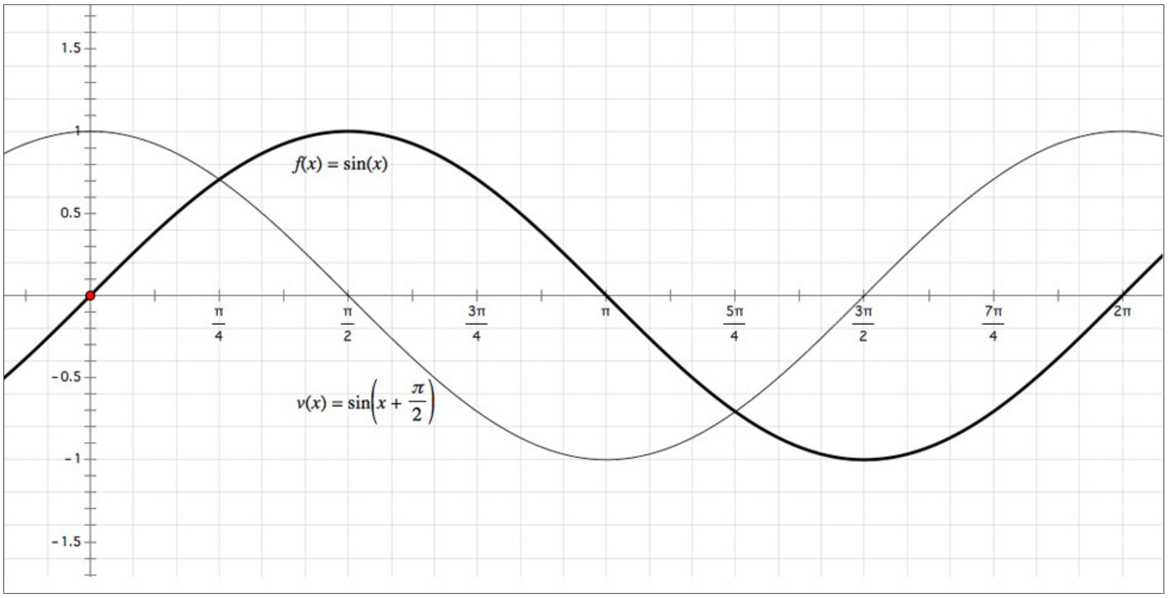
2. Graph $y = \sin(x - \frac{\pi}{2})$ onto the same set of axes as the parent function and label it.

   Solution:



3. Graph $y = \sin(x + \frac{\pi}{2})$ onto the same set of axes and label it. (You'll have three graphs on one set of axes.).

Solution:



4. What can you conclude about the way the value of *C* in the equation above affects the graph of the basic sine function? In general, what effect does subtracting a constant from the independent variable have on the parent function's graph? Use additional evidence from Handout 5 to back up your conclusion.

Answer: When a constant is subtracted from the independent variable, the function moves to the right if the constant is positive and to the left if the constant is negative. This is also true in general. The graphs in problem 3 from Handout 5 show the same pattern.

5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in *C* might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

Answer: No change will happen—the numerical location of the crests and troughs are arbitrary and will not affect the sound.

Part 4: The effect of *D* in *y* = sin(*x*) + *D*

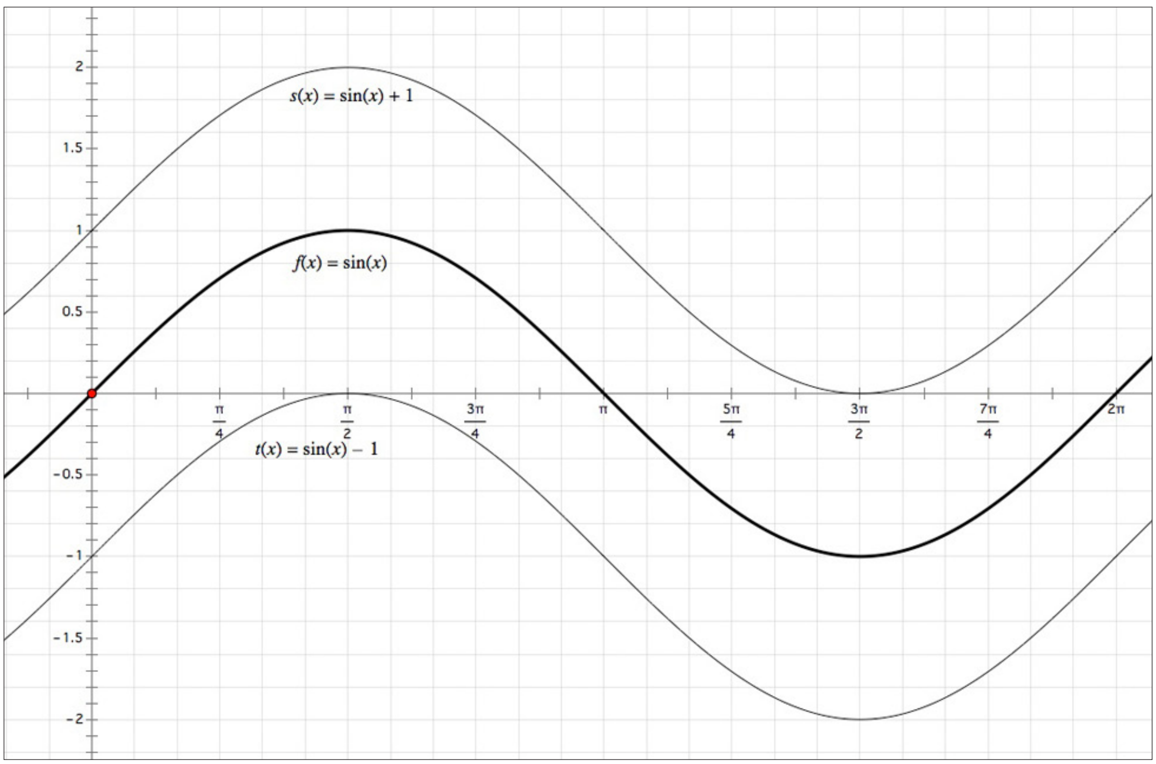
Stretch the graph of the parent function $y = \sin(x)$ and explore the effect of changing the value of D in the equation $y = \sin(x) + D$.

1. Sketch the graph of $y = \sin(x)$ on graph paper.

   Solution: *See the graph on page 80.*

2. Graph $y = \sin(x) + 1$ and $y = \sin(x) - 1$ onto the same set of axes as the parent function and label them. (You'll have three graphs on one set of axes.)

   Solution:



3. What can you conclude about the way the value of D in the equation above affects the graph of the basic sine function? In general, what effect does adding a constant to a function have on the function's graph? Use additional evidence from Handout 5 to back up your conclusion.

   Answer: *The value of D moves the graph up (if $D > 0$) or down (if $D < 0$). In general, adding a constant to a function will move the function up or down. The graphs in problem 1 from Handout 5 show the same pattern, depending on the sign of the constant.*

Part 5: Understanding the general sine function

A sine function has the form $y = A \sin(Bx - C) + D$.

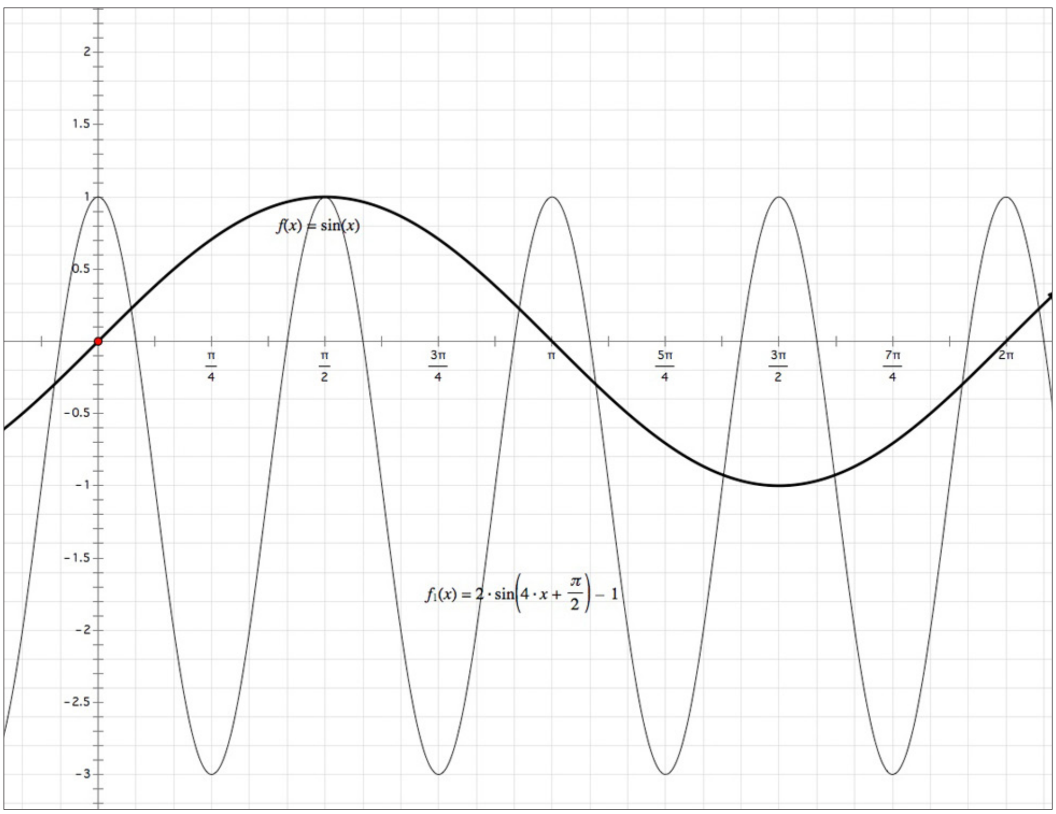
Use your understanding of function transformations to generate the graph of the following sine function:

$$y = 2 \sin(4x + \frac{\pi}{2}) - 1$$

Before creating this graph, predict how the value of each constant in the function will affect the graph of $y = \sin x$.

In the next activity, you'll have further opportunities to make connections between the transformed graphs and the effects of transformations on the sounds they represent.

Solution:



Note: The phase shift is not $\frac{\pi}{2}$ in this case, but $\frac{\pi}{8}$. The variable is multiplied before the constant is added.

Part 3: Generating Sound

Students use sound editing software to turn sine equations into sounds. They begin by determining the period, frequency, and amplitude of the waves represented by the equation. Next, they compare equations and their resulting sounds, making predictions about how the sounds and graphs will be different. Finally, they experience how changes in the waveforms affect the sounds they hear.

Length

1 50-minute session

Advance Preparation

- Reserve a computer lab and install sound editing software on the computers. (See *Media & Resources* for a link to the recommended software.)

RESOURCES

> **Note:** *Media & Resources* includes a link to Audacity, a free, downloadable sound editing program. If your school has already installed a different sound editing program in its computer lab, test that program in advance. You may want to ask the computer or audio teacher for a quick demonstration. You'll also need to modify **Handout 7: Sound Equations** with instructions for the program you are using.

- If you choose to use optional **Handout 8: Note Frequencies**, decide whether you'll have students use a decibel monitor, and procure one if needed.

Understandings

- The pitch of a sound is altered by changing the frequency (that is, by a horizontal stretch of the mathematical model).
- The volume of a sound is altered by changing the amplitude (that is, by a vertical stretch of the mathematical model).

Materials Needed

- **Handout 7: Sound Equations**
- **Assessment Checklist: Generating Sound**
- Computer with sound editing software (one for each pair of students) (see *Advance Preparation*)
- Optional: Headphones
- Optional: Decibel monitor (one for students to share)
- Optional: **Handout 8: Note Frequencies**

1. Have students make predictions about sounds and graphs.
Distribute **Handout 7: Sound Equations** and **Assessment Checklist: Generating Sound**. Answer any questions students may have about how their work will be assessed. Have students work on Part 1 of Handout 7, comparing each pair of equations. They should describe how the sounds will be different; for the graphs, they can describe the difference in words or make sketches.

2. Have students work in pairs to check their predictions and create sounds.
When students have written or sketched their predictions for Part 1, have them work in pairs at a computer. Show them how to open and use the sound editing software.

Have them complete Part 2 of Handout 7, completing the table, checking their answers, and generating audio files. As students complete the table with amplitude, period, and frequency, you may need to remind them how to find each of these values.

> **Note:** You may want to have headphones available, especially if the computers are close together.

If students finish early, ask them not to start working on Part 3 of Handout 7. Instead, you can give them the chart of frequencies on **Handout 8: Note Frequencies** to try composing a song (or recreating a simple one, such as "Mary Had a Little Lamb"), using the sound editing software. You may need to show them how to use the cursor tool to select the point at which a tone (or silence) should be generated.

Teacher's Notes: Sound Values and Volume

If students try additional values on their own, they may think that greater values of *B* also provide greater volume. The issue here is that the human ear is more sensitive to high pitches, so these can be interpreted as being louder. (This is why small dogs with high-pitched barks seem to produce more noise than larger dogs with lower barks, especially from a distance.) The values in the problems were chosen to minimize this perception.

If you have access to a decibel monitor, you might have students reproduce the sounds and use the monitor to measure their actual volume.

3. Summarize the effects of the multipliers on the sound.

Discuss students' results—how did their predictions compare to the actual sounds and graphs? Revisit the physical meaning of the values, and make sure that students understand the following about the general sine equation:

- Values for *A* affect the *amplitude* of the wave.
- On the graph, the distance between a crest and a trough is greater when *A* is greater.
- The sound is louder when *A* is greater, which means that the compression of the air molecules is tighter (more compressed), and the rarefaction is looser (more open).
- Values for *B* affect the *period* and *frequency* of the wave.
- The graph is narrower when *B* is greater.
- The sound has a higher pitch when *B* is greater, which means that the object causing the sound is vibrating faster.

Ask leading questions, if necessary, and encourage every student to add to the discussion, so you can check each student's understanding.

Have students complete Part 3 of Handout 7 individually.

Have students complete the student comments section of **Assessment Checklist: Generating Sound**.

Teacher's Notes: Other Sine Equations

You might want to discuss the fact that in these examples, there are no values being added—both *C* and *D* from the general equation $y = A \sin B(x - C) + D$ are equal to 0. In Audacity, there is no way to include these values; since they are irrelevant to the sound that would be generated, there's no need to include them.

Handout 7:
Sound Equations

Each equation in this handout represents a sound. The variable x represents time in seconds, and y represents air pressure.

Part 1: Predicting effects

For each of the following, use the values in the equations to predict how the sound and graph represented by the *first* equation will be different from the sound and graph represented by the *second* equation. For example, will it be louder, softer, or the same volume? How will the graph look different?

1. Compare $y = 0.5 \sin 900x$ and $y = 0.2 \sin 900x$:

   How will the sound generated by $y = 0.5 \sin 900x$ be different from the sound generated by $y = 0.2 \sin 900x$?

   Answer: y = 0.5 sin 900x will be louder than y = 0.2 sin 900x.

   How will the graph of $y = 0.5 \sin 900x$ be different from the graph of $y = 0.2 \sin 900x$?

   Answer: The graph of y = 0.5 sin 900x will have higher crests and deeper troughs than the graph of y = 0.2 sin 900x (that is, there will be greater vertical distance between the crests and troughs).

2. Compare $y = 0.5 \sin 900x$ and $y = 0.85 \sin 900x$:

   How will the sound generated by $y = 0.5 \sin 900x$ be different from the sound generated by $y = 0.85 \sin 900x$?

   Answer: y = 0.5 sin 900x will be quieter than y = 0.85 sin 900x.

   How will the graph of $y = 0.5 \sin 900x$ be different from the graph of $y = 0.85 \sin 900x$?

   Answer: The graph of y = 0.5 sin 900x will have lower crests and higher troughs than the graph of y = 0.85 sin 900x (that is, there will be less vertical distance between the crests and troughs).

3. Compare $y = 0.85 \sin 900x$ and $y = 0.85 \sin 1000x$:

   How will the sound generated by $y = 0.85 \sin 900x$ be different from the sound generated by $y = 0.85 \sin 1000x$?

   Answer: y = 0.85 sin 900x will have a lower pitch than y = 0.85 sin 1000x.

   How will the graph of $y = 0.85 \sin 900x$ be different from the graph of $y = 0.85 \sin 1000x$?

   Answer: The graph of y = 0.85 sin 900x will have a wider wave than the graph of y = 0.85 sin 1000x (that is, there will be greater horizontal distance between crests or between troughs, or even between a crest and the next trough).

4. Compare $y = 0.85 \sin 900x$ and $y = 0.85 \sin 800x$:

   How will the sound generated by $y = 0.85 \sin 900x$ be different from the sound generated by $y = 0.85 \sin 800x$?

   ***Answer:* y = 0.85 sin 900x *will have a higher pitch than* y = 0.85 sin 800x.**

   How will the graph of $y = 0.85 \sin 900x$ be different from the graph of $y = 0.85 \sin 800x$?

   Answer: The graph of* y = 0.85 sin 900x *will have a narrower wave than the graph of* y = 0.85 sin 800x *(that is, there will be less horizontal distance between crests or between troughs, or even between a crest and the next trough).

5. Compare $y = 0.85 \sin 800x$ and $y = 0.2 \sin 900x$:

   How will the sound generated by $y = 0.85 \sin 800x$ be different from the sound generated by $y = 0.2 \sin 900x$?

   ***Answer:* y = 0.85 sin 800x *will have a lower pitch and be louder than* y = 0.2 sin 900x.**

   How will the graph of $y = 0.85 \sin 800x$ be different from the graph of $y = 0.2 \sin 900x$?

   Answer: The graph of* y = 0.85 sin 800x *will have a wider wave than the graph of* y = 0.2 sin 900x*, with higher crests and deeper troughs (that is, there will be greater horizontal distance between crests or between troughs, or even between a crest and the next trough, and greater vertical distance between crests and troughs).

When you have completed these problems, meet with a partner to test your predictions.

Part 2: Testing your predictions

You'll now use sound editing software to generate audio files of these equations so you can compare them.

- Complete the following table with your partner, identifying the amplitude, period, and frequency for each equation. The first one is completed for you.

Equation	Amplitude	Period (in seconds)	Frequency (hertz, to nearest thousandth)
$y = 0.5 \sin 900x$	0.5	$\frac{\pi}{450}$ or about 0.00698	143.239
$y = 0.2 \sin 900x$	*0.2*	$\frac{\pi}{450}$ *or about 0.00698*	*143.239*
$y = 0.85 \sin 900x$	*0.85*	$\frac{\pi}{450}$ *or about 0.00698*	*143.239*
$y = 0.85 \sin 800x$	*0.85*	$\frac{\pi}{400}$ *or about 0.00785*	*127.324*
$y = 0.85 \sin 1000x$	*0.85*	$\frac{\pi}{500}$ *or about 0.00628*	*159.155*

- Check your answers to the first problem (comparing $y = 0.5 \sin 900x$ and $y = 0.2 \sin 900x$) against your partner's and discuss any disagreements you may have.

- Open the software application and do the following to generate a tone for $y = 0.5 \sin 900x$:
  - From the menu bar, choose **Generate > Tone . . . .**
  - Be sure the Waveform is set for **Sine**, and then enter the frequency and amplitude for $y = 0.5 \sin 900x$ from the table. Click OK.
  - Select the Zoom tool (magnifying glass). Click on the band representing the sound multiple times, until you see the thousandths place in the numbers (for example, 2.230, 2.240, 2.250, . . .). You will gradually be able to see the sound wave.
  - From the menu bar, choose **Save Project** and give it a name that will help you remember which equation was used. (For example, "Apoint5 B900" might work.) This will let you reuse the tone later rather than have to generate it again.

- Do the following to generate a tone for $y = 0.2 \sin 900x$:
  - From the menu bar, choose **File > New**. (This will prevent you from accidentally overwriting the other tone.)
  - Choose **Generate > Tone . . .** and enter the frequency and amplitude for the new equation. Click OK.

- Again, use the Zoom tool to zoom in until you see the thousandths place in the numbers. *Be sure both graphs have the same scales*, even if you are not looking at exactly the same time interval.
- Save the project with a name that helps you remember which equation was used. (For example, "Apoint2 B900" might work.)

- Compare the tones and graphs:
  - Play each tone—first one, then the other—by clicking the green arrow in each window. Was your prediction about how they would be different correct?
  - Now look at the graphs generated for each tone. Was your prediction about how they would be different correct?

- Use the same process to compare the other pairs (you don't need to generate a tone you've already generated, if you can remember which window belongs with which equation):
  - $y = 0.5 \sin 900x$ and $y = 0.85 \sin 900x$
  - $y = 0.85 \sin 900x$ and $y = 0.85 \sin 1000x$
  - $y = 0.85 \sin 900x$ and $y = 0.85 \sin 800x$
  - $y = 0.85 \sin 800x$ and $y = 0.2 \sin 900x$

  Did you correctly predict how the sounds and graphs of these pairs would be different?

Part 3: Conclusions

Summarize the effects of the multipliers *A* and *B* in the equation $y = A\sin(Bx)$ on the resulting sound by completing the following:

1. When the value of A increases, the sound ____________________ because ________________________

   ____

   ____

   Possible answer: . . . gets louder, because A controls the amplitude, that is, how intense the compression and rarefaction are.

2. When the value of B increases, the sound ____________________ because ________________________

   ____

   ____

   Possible answer: . . . has a higher pitch, because B controls the frequency, that is, the speed of the vibrations that create the sound.

Assessment Checklist:
Generating Sound

Use this assessment to help you as you connect transformations of the sine equation to changes in the sound it represents. Make sure to include all the requirements. Your teacher will use this assessment to evaluate your work.

Requirements	Suggested Percentage of Total Grade	Comments	
Generating Sounds		**Student Comments**	**Teacher Comments**
Made predictions on Handout 7 before testing.	20%		
Correctly found the amplitude and period of each equation in Part 2 of Handout 7, and calculated the corresponding frequency.	35%		
Participated in class discussions.	10%		
Provided a correct conclusion for both multipliers on Handout 7, including reasoning why each has the effect it has.	35%		
Total	**100%**		

Handout 8:
Note Frequencies

The table below provides the frequencies for some musical notes. Notice that the frequency of the same note, but an octave higher, is doubled. You can use this relationship to find frequencies for other octaves, if needed.

You can use these frequencies in sound editing software to create your own song or to recreate one you're already familiar with. In Audacity, you can use the cursor tool to indicate where you want a tone or silence to be generated, or to select a region so you can move it.

For example, "Mary Had a Little Lamb" can be recreated using the notes C, D, E, and G:

E D C D E E E (rest or silence) D D D (rest) D G G (rest) E D C D E E E E D D E D C

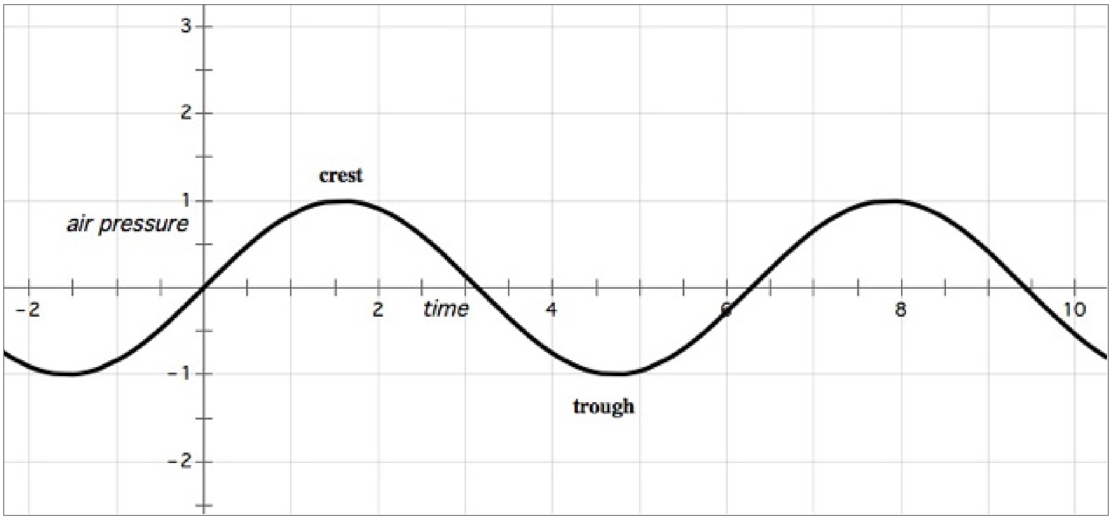
You'll need to think about how much time (duration) to give each tone, and put in a small amount of silence between notes to separate them.

Note Frequencies

Note	Frequency (Hz)
A	110.00
$A^{\#}/B^{b}$	116.54
B	123.47
C (middle C)	130.81
$C^{\#}/D^{b}$	138.59
D	146.83
$D^{\#}/E^{b}$	155.65
E	164.81
F	174.61
$F^{\#}/G^{b}$	185.00
G	196.00
$G^{\#}/A^{b}$	207.65

Note	Frequency (Hz)
A	220.00
$A^{\#}/B^{b}$	233.08
B	246.94
C	261.63
$C^{\#}/D^{b}$	277.18
D	293.66
$D^{\#}/E^{b}$	311.13
E	329.63
F	349.23
$F^{\#}/G^{b}$	369.99
G	392.00
$G^{\#}/A^{b}$	415.30

Appendix A: Sound Wave Image



Materials Needed

Throughout Unit

- Chart paper and markers

Part 1: Making Waves

Supplies and Equipment

- Materials for the sound demonstration you will perform (see *Advance Preparation*)
- Computers with Internet access and speakers (one per pair of students, or one for the whole class with a projector)
- Computers (one per pair of students, or one for the whole class with a projector) with Internet access (see *Advance Preparation*)
- Graph paper
- Colored pencils
- Graphing calculators
- Rulers
- Protractors

Handouts

- **Handout 1: Exploring Sound**
- **Handout 2: Unit Overview**
- **Handout 3: Finding the Wave Function**
- **Handout 4: The Graph of Cosine**

Examples of Media Resources

- Projection or printout of the graph in **Appendix A: Sound Wave Image**
- Waveform applet (see *Advance Preparation*)

Advance Preparation

- Before Activity 1A, gather materials you can use to demonstrate for students how sound is produced by vibrations from an object. Ideally, use objects that vibrate in a noticeable way—for example, pluck a guitar string or strike a tuning fork.
- Before Activity 1A, review **Handout 1: Exploring Sound** and the applets and simulations on the Web sites listed in the handout. See *Media & Resources* for more information about these Web sites.
- Before Activity 1B, preview the waveform applet, available online. See *Media & Resources* for a link to this applet.

Part 2: Making Changes

Supplies and Equipment

- Graphing calculators
- Graph paper

Handouts

- **Handout 5A–D: Function Transformations Group 1–4** (one copy of each part for one-fourth of the class)
- **Handout 6: Changing Sine**

Items Students Need to Bring

- Copies of **Handout 1: Exploring Sound**

Part 3: Generating Sound

Supplies and Other Equipment

- Computer with sound editing software (one for each pair of students) (see *Advance Preparation*)
- Optional: Headphones
- Optional: Decibel monitor (one for students to share)

Handouts

- **Handout 7: Sound Equations**
- **Assessment Checklist: Generating Sound**
- Optional: **Handout 8: Note Frequencies**

Advance Preparation

- Reserve a computer lab and install sound editing software on the computers. (See *Media & Resources* for a link to the recommended software.)

  Note: *Media & Resources* includes a link to Audacity, a free, downloadable sound editing program. If your school has already installed a different sound editing program in its computer lab, test that program in advance. You may want to ask the computer or audio teacher for a quick demonstration. You'll also need to modify **Handout 7: Sound Equations** with instructions for the program you are using.

- If you choose to use optional **Handout 8: Note Frequencies**, decide whether you will have students use a decibel monitor, and procure one if needed.

Media & Resources

These recommended Web sites have been checked for availability and for advertising and other inappropriate content. However, because Web site policies and content change frequently, we suggest that you preview the sites shortly before using them.

Media & Resources are also available at http://dma.edc.org and at http://dmamediaandresources.pbworks.com, a Wiki that allows users to add and edit content.

Part 1: Making Waves

Activity 1A: Exploring Sound

Illuminations Sound Wave Applet

The sound wave applet is part of the National Council of Teachers of Mathematics *Illuminations: Resources for Teaching Math* Web site.

http://illuminations.nctm.org/tools/soundwave/soundwave.html

More information about the applet can be found here: http://illuminations.nctm.org/ActivityDetail.aspx?id=37. If the URL has changed, you should be able to locate the applet by searching for "NCTM illuminations sound wave applet."

Sounds Amazing

This Web site about sound was developed by the University of Salford, Manchester, United Kingdom. If the URL has changed, you should be able to locate the applet by searching for "Sounds Amazing Web site University of Salford."

www.acoustics.salford.ac.uk/schools/index1.htm

Activity 1B: Generating Sine and Cosine Graphs

For a link to the waveform applet, go to the *Media & Resources* for Functions and Sound on the DMA Web site: dma.edc.org/unit/functions-and-sound-algebra-2. The applet runs within a browser window.

Part 3: Generating Sound

Sound Editing Software

Audacity

Free sound editing software for PC and Mac

audacity.sourceforge.net

Standards

This unit was developed to meet the following standards.

California Academic Content Standards for Mathematics, Grades 9–12

- Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression. *[Algebra 1, 17.0]*
- Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x - b)^2 + c$. *[Algebra 2, 9.0]*

CTE AME Industry Sector Foundation Standards

4.0 Technology

Students know how to use contemporary and emerging technological resources in diverse and changing personal, community, and workplace environments:

> **4.2** Understand the use of technological resources to gain access to, manipulate, and produce information, products, and services.
> **4.7** Understand how technology can reinforce, enhance, or alter products and performances.

11.0 Demonstration

Students demonstrate and apply the concepts contained in the foundation and pathway standards.

NCTM Standards

- Students understand patterns, relations, and functions, and select, convert flexibly among, and use various representations for them. *[Algebra]*
- Students use symbolic algebra to represent and explain mathematical relationships. *[Algebra]*
- Students understand and compare properties of classes of functions, including exponential, polynomial, and periodic functions. *[Algebra]*
- Students identify essential quantitative relationships in a situation and determine the class of functions that might model the relationships. *[Algebra]*
- Students solve problems that arise in mathematics and other contexts. *[Problem Solving]*

- Students use the language of mathematics to express mathematical ideas precisely. *[Communication]*
- Students use representations to model and interpret physical, social, and mathematical phenomena. *[Representation]*
- Students recognize and use connections among mathematical ideas. *[Connections]*