

## Handout 1: Exploring Sound

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Using computer simulations, you will explore the nature of sound as it travels through air. In particular, you'll consider what happens when the *pitch* of a sound changes and when the *volume* of a sound changes.

### Exploration 1: Illuminations Sound Wave

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Open the Sound Wave applet (your teacher will give you the URL).

There are two parameters that can be changed: the *initial string displacement* and the *tension* in the string. (Imagine that the string is a guitar string or one of a piano's strings.) The goal for this exploration is to determine how changes in these parameters affect the sound that is produced.

#### Step 1

Set the initial string displacement at a low value by moving the slider to the left. Click the "hear" button to hear the sound. Move the Initial String Displacement slider toward the right.

Write a description of how increasing the initial string displacement affects the sound. (Click the "hear" button again to stop the sound while you write your description.)

#### Step 2

Move the Initial String Displacement slider to a middle value. Click the "start" button to start the string vibrating again. You will see dots that represent air molecules being moved by the string.

Describe the pattern you see for this single value of initial string displacement.

### Step 3

Move the Initial String Displacement slider and notice how the pattern changes. You may want to compare a very low value (but not 0), a very high value, and a value in the middle.

Describe the change in the pattern.

### Step 4

What do you think this change in pattern means for the air molecules? Write about how changing the Initial String Displacement changes the way the string vibrates.

### Step 5

Move the Initial String Displacement slider to a high value. Press the "hear" button and move the String Tension slider. Listen to the changes in the sound produced.

Describe the effect of increasing the string tension on the sound that you hear.

### Step 6

Try this again, using the "start" button to see the representation of the air molecules. Move the String Tension slider and notice how the pattern changes.

Describe the effect of increasing the string tension on the pattern.

## Exploration 2: Sounds Amazing Simulation

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Go to the Sounds Amazing Web site (your teacher will give you the URL).

Click on “What is Sound?” on the left.

Read each slide and click the buttons for the simulations. When you’re done with a slide, click the right arrow below it to go to the next page. (Note: Some slides have their own “next” arrow on them. Use those first.)

**Stop when you’re done with page 5, Loudness and Decibels.**

Take notes on any vocabulary words and their meanings, and any other information you think is important.

Answer these questions:

- What do you notice about the relationship between pitch and the shape of the wave produced by a particular sound?
  
- What do you notice about the relationship between volume and the shape of the wave produced by a particular sound?

## Summary: The Nature of Sound

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Record what you’ve learned about sound from your explorations and the class discussion. Take good notes—you’ll need to refer to these ideas again later!

- What did you learn about how sound is created?
  
- What seems to affect the volume of a sound?
  
- What seems to affect the *pitch* (whether a tone is high or low) of a sound?

## Handout 2: Unit Overview

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*In this unit, you'll explore how sound works—from how it is created, to what affects its pitch and volume. You'll connect the physical properties of sound to the way it moves through the air, using mathematics to create graphs depicting sound waves. You'll also examine how changing the constants and coefficients in an equation can affect the graph of that equation, and ultimately how this can be used to create different tones.*

*Your work in this unit will revolve around the following questions:*

- *How can we mathematically represent an invisible phenomenon, such as sound?*
- *How are changes in a mathematical equation reflected in its graph?*
- *How can the volume and pitch of a sound be altered through changes in the function that represents that sound?*

### Unit Project

Once you've explored how sound can be modeled with waves, and how changing the parameters of an equation can affect its graph, you'll work with a partner to predict how the parameters in different equations will produce different waves—and therefore different sounds—focusing on both pitch and volume. You'll then check your predictions, using sound editing software. You may even choose to use this method to recreate a simple tune or to compose your own original musical work.

### What You Will Do in This Unit

**Explore how sound is created.** Use computer applets to help you visualize how sounds are created and transmitted and how this process can be translated into a wave.

**Give sound a mathematical context.** Use your knowledge of right-angle trigonometry, and how it connects to circular functions and the unit circle, to create the graphs of sine and cosine functions.

**Explore the effects of changing parameters on graphs.** Change coefficients (multipliers) and add constants to several basic equations and compare the resulting graphs. Predict how changing the parameters of a function will affect its graph.

**Manipulate waves.** Apply what you discovered about parameter changes to sine functions, and connect this knowledge to how changes in parameters would affect the sounds modeled by sine functions.

**Create sounds.** Predict how sounds with different wave models will compare to each other, and test your predictions by using tools to produce sounds with given parameters.

## Vocabulary Used in This Unit

**Amplitude:** (1) Half the difference between the maximum and minimum y-values of a sine or cosine wave. (2) The intensity of the air pressure caused by the vibrations that created the sound. This air pressure is measured in Newtons/meter<sup>2</sup> or decibels (dB), a logarithmic unit. Amplitude is related to the volume we perceive.

**Compression:** The crowding together of air molecules.

**Cosine:** A trigonometric function, using an angle measure as the independent variable. Cosine is defined using acute angles in right triangles as the ratio of the length of the adjacent leg to the length of the hypotenuse. It's represented in function notation as  $\cos(x)$ .

**Crest:** The point at which a wave reaches its highest value.

**Cyclical:** Recurring over and over at regular intervals, just as tracing a circle returns one to the same place over and over. Also referred to as *periodic* in mathematics.

**Decompression (or rarefaction):** Air molecules that are spread out.

**Frequency of a sound wave:** The speed at which the air pressure changes at a given point in space. The frequency of a wave is the number of cycles per unit time, or the number of cycles per second. Hertz (Hz) is the unit used to measure sound frequency and is defined as 1 cycle/second. Frequency is related to pitch in human perception. For example, a sound wave of 440 Hz sounds like the note A on a piano (just above middle C). Humans hear in a frequency range of about 20 Hz to 20,000 Hz.

**Period of a sound wave:** The time it takes to complete one cycle, measured in seconds/cycle. Frequency and period have an inverse relationship. If the x-axis represents time, the period is also the horizontal distance between consecutive crests (or troughs) in a sine or cosine wave.

**Phase of a wave:** The horizontal offset from the normal starting position of the basic wave  $y = \sin x$  or  $y = \cos x$  at  $x = 0$ .

**Radian:** A unit of measurement used for cyclical phenomena so that angles may be treated as distances. In a *unit circle* (a circle whose radius is 1 unit), a central angle that intercepts an arc whose length is 1 unit measures 1 radian.

**Sine:** A trigonometric function, using an angle measure as the independent variable. Sine is defined using acute angles in right triangles as the ratio of the length of the opposite leg to the length of the hypotenuse. It's represented in function notation as  $\sin(x)$ .

**Sinusoidal:** A graph or phenomenon that takes the shape of a sine wave, oscillating up and down in a regular, continuous manner.

**Sound synthesis:** This occurs when the changes that different sounds make in air pressure amplitude are combined and blended into one sound. When two sound waves travel through the same air space at the same time, their amplitudes at each moment in time are summed into a composite wave that contains the frequencies of each.

With an audio editing program, separate waves can be shown in different tracks. The editing software can then combine the waves into a single sound wave that has the frequency components of each wave.

**Sound wave propagation:** The passing along of the change in air pressure from air molecules to adjacent air molecules.

**Trough:** The point at which a wave reaches its lowest value.

## Handout 3: Finding the Wave Function

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The steps below will help you find a mathematical function used to represent sound waves. Follow the instructions and write down your answers to each question.

### Step 1: Traveling a circle

Look at the waveform applet, which shows a circle on a coordinate grid. At the bottom of the applet you'll see a line and a dot labeled "time = 0."

- Click on the time dot and slowly drag it to the right, so time increases from 0 to 10. Notice that the red dot travels around the circle.
- At what time value does the red dot return to its original position on the circle?
  
- Imagine that you can keep increasing the time, past time = 10. When do you think the red dot would return again to its original position?

The red dot's travel around the circle is *cyclical*, just as a wave is cyclical, because it keeps repeating its path, always returning to where it's been before. Circles and wheels are often used (in engines, for example) to create an ongoing, cyclical behavior.

### Step 2: Graphing the wave

Stay with the waveform applet, returning the time dot to the time = 0 position.

- Click the box next to "Graph the wave" so a check appears in the box.
- Once again, click and slowly drag the time dot to the right. This time the circle will move so that the dot's x value is always the same as the time value. The dot will leave images of itself as it moves.
- At what time does the red dot hit the first *crest* (the high point of the wave)?
  
- Where is the dot at about twice that time (twice the time it took to hit the first crest)?
  
  
- Where is the dot at about three times the time it took to hit the first crest?

- Where is the dot at about four times the time it took to hit the first crest?

The wave created by the dot traveling around the circle looks a lot like a sound wave. If you could keep moving the time dot past 10, it would continue to make the same wave over and over again.

### Step 3: Finding a function

Now that you can see a way to create a graph that looks like the wave, try to find a mathematical function that can be used to represent that graph. Stay with the waveform applet, keeping the box next to “Graph the wave” checked.

- Click the box next to “Show height” so that a check appears in the box.
- Move the time dot so that you can clearly see a right triangle inside the circle. (The exact time value doesn’t matter, just find one for which you can comfortably see the triangle.)
- Sketch the circle and triangle below, just as they appear on your screen.

- What is the length of the hypotenuse? Label your sketch with that length.
- The height of the triangle is the  $y$  value of the red dot—that is, it’s the value of the function for that time value. With your partner, consider what functions you might know from past work that express ratios comparing the sides of right triangles. (Hint: The functions use an *angle* in the triangle.) Be prepared to discuss your ideas about what functions are helpful.



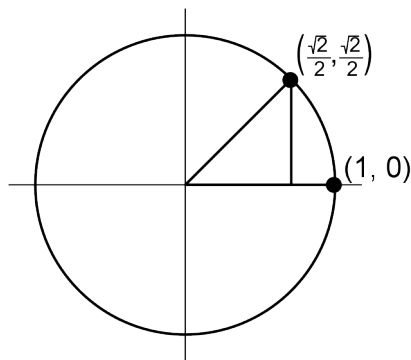


## Handout 4: The Graph of Cosine

As you've seen, the sine and cosine functions can be used to model sound waves. To help you better understand the cyclical nature of these functions, you will generate the graph of the cosine function using right triangles and a unit circle.

### Part 1: Generating Cosine

- Start by drawing a right triangle in the unit circle shown below. The unit circle has radius 1. In the diagram, a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle has been drawn in the unit circle, with the vertex of one of the acute angles at the center of the circle.



- Following this example, use a protractor to draw right triangles inside the unit circle, measuring angles in the counterclockwise direction. Use an increment of  $45^\circ$  as you increase angle measures. Note that at  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , and  $360^\circ$ , you will be unable to sketch right triangles. For these angles, mark the coordinates of the point on the circle (e.g., at  $0^\circ$ , the coordinates of the point on the circle where the initial side of the triangle and the terminal side of the triangle coincide are  $[1, 0]$ ).
- Complete the Table of Cosine Values below, using the definition of the cosine of an angle, the ratio of the adjacent side of the angle, and the hypotenuse of the right triangle. (The first entry is already completed.)

Notes:

- The position of the angle drawn in the unit circle matters when determining the **sign** of the value of the cosine of the angle.
- Use your knowledge of special triangles to determine the cosine of multiples of  $45^\circ$ .
- Use the **symmetry** of the circle to determine the cosine of multiples of  $45^\circ$ .

Table of Cosine Values

Acute angle measure, $\phi$ (in degrees)	Acute angle measure, $\phi$ (in radians)	$\cos \phi$	Coordinates of the point on the unit circle
$0^\circ$	0 radians	1	(1, 0)
$45^\circ$			
$90^\circ$			
$135^\circ$			
$180^\circ$			
$225^\circ$			
$270^\circ$			
$315^\circ$			
$360^\circ$			

Once you have completed the Table of Cosine Values, transfer these values to the coordinate system below. Notice that the x-axis is labeled using radian measure and that the y-axis values range from  $-1$  to  $1$ .



3. How might you change the exercise above to create a graph of a sine wave that ranges between  $-2$  and  $2$ ? Try doing this on a separate piece of graph paper.

4. How could you generate additional values that would yield a closer approximation to a waveform?

5. Did generating the basic cosine wave using the unit circle lead you to any new ideas or discoveries?

## Handout 5A: Function Transformations Group 1

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### Parent function: $y = x^2$

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In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you will use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #1, will work on altering the quadratic equation whose graph is the basic parabola. Use separate sheets of graph paper for your work.

#### Step 1

Each group member should sketch a copy of the function  $y = x^2$ . (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

#### Step 2

The following problems make four different kinds of changes to the original parent equation of the parabola you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

##### Problem 1

Graph the following equations on the same axes as the parent function  $y = x^2$ :

$$y = x^2 + 3 \qquad y = -2 + x^2$$

Generalize your observations by predicting what the graph of  $y = x^2 + c$  will look like in comparison to the parent function  $y = x^2$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = x^2 - \left(\frac{1}{2}\right)$  will look like, and then graph it.

**Problem 2**

Graph the following equations on the same axes as the parent function  $y = x^2$ :

$$y = -3x^2 \qquad y = \left(\frac{1}{2}\right)x^2$$

Generalize your observations by predicting what the graph of  $y = cx^2$  will look like in comparison to the parent function  $y = x^2$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = -\frac{1}{4}x^2$  will look like, and then graph it.

**Problem 3**

Graph the following equations on the same axes as the parent function  $y = x^2$ :

$$y = (x + 5)^2 \qquad y = (x - 3)^2$$

Generalize your observations by predicting what the graph of  $y = (x - c)^2$  will look like in comparison to the parent function  $y = x^2$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = (x - \frac{1}{2})^2$  will look like, and then graph it.

**Problem 4**

Graph the following equations on the same axes as the parent function  $y = x^2$ :

$$y = (2x)^2 \qquad y = \left(\frac{1}{2}x\right)^2$$

Generalize your observations by predicting what the graph of  $y = (cx)^2$  will look like in comparison to the parent function  $y = x^2$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = (4x)^2$  will look like, and then graph it.

**Step 3**

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function  $y = x^2$ :

$$y = 3(x + 5)^2 \qquad y = -2(x - 3)^2 + 1$$

After you've recorded your predictions, test them by graphing each equation.

## Handout 5B: Function Transformations Group 2

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### Parent function: $y = x^3$

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In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #2, will work on altering the quadratic equation whose graph is the basic cubic function. Use separate sheets of graph paper for your work.

#### Step 1

Each group member should sketch a copy of the function  $y = x^3$ . (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

#### Step 2

The following problems make four different kinds of changes to the original parent equation of the cubic function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

##### Problem 1

Graph the following equations on the same axes as the parent function  $y = x^3$ :

$$y = x^3 + 4 \qquad y = -1 + x^3$$

Generalize your observations by predicting what the graph of  $y = x^3 + c$  will look like in comparison to the parent function  $y = x^3$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = x^3 - \frac{1}{2}$  will look like, and then graph it.

**Problem 2**

Graph the following equations on the same axes as the parent function  $y = x^3$ :

$$y = -3x^3 \quad y = \frac{1}{2}x^3$$

Generalize your observations by predicting what the graph of  $y = cx^3$  will look like in comparison to the parent function  $y = x^3$ , no matter what value  $c$  has. Test your idea by predicting what the graph of

$y = -\frac{1}{4}x^3$  will look like, and then graph it.

**Problem 3**

Graph the following equations on the same axes as the parent function  $y = x^3$ :

$$y = (x + 2)^3 \quad y = (x - 5)^3$$

Generalize your observations by predicting what the graph of  $y = (x - c)^3$  will look like in comparison to the parent function  $y = x^3$ , no matter what value  $c$  has. Test your idea by predicting what the graph of

$y = (x - \frac{1}{2})^3$  will look like, and then graph it.

**Problem 4**

Graph the following equations on the same axes as the parent function  $y = x^3$ :

$$y = (2x)^3 \quad y = (\frac{1}{2}x)^3$$

Generalize your observations by predicting what the graph of  $y = (cx)^3$  will look like in comparison to the parent function  $y = x^3$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = (4x)^3$  will look like, and then graph it.

**Step 3**

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function  $y = x^3$ :

$$y = 3(x + 5)^3 \quad y = -2(x - 3)^3 + 1$$

After you've recorded your predictions, test them by graphing each equation.



## Handout 5C: Function Transformations Group 3

### Parent function: $y = 3^x$

In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #3, will work on altering the quadratic equation whose graph is the basic exponential function. Use separate sheets of graph paper for your work.

#### Step 1

Each member of the group should sketch a copy of the function  $y = 3^x$ . (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

#### Step 2

The following problems make four different kinds of changes to the original parent equation of the exponential function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

##### Problem 1

Graph the following equations on the same axes as the parent function  $y = 3^x$ :

$$y = 3^x + 4 \qquad y = -1 + 3^x$$

Generalize your observations by predicting what the graph of  $y = 3^x + c$  will look like in comparison to the parent function  $y = 3^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = 3^x - \frac{1}{2}$  will look like, and then graph it.

**Problem 2**

Graph the following equations on the same axes as the parent function  $y = 3^x$ :

$$y = -3(3^x) \quad y = \frac{1}{2}(3^x)$$

Generalize your observations by predicting what the graph of  $y = c3^x$  will look like in comparison to the parent function  $y = 3^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = -\frac{1}{4}(3^x)$  will look like, and then graph it.

**Problem 3**

Graph the following equations on the same axes as the parent function  $y = 3^x$ :

$$y = 3^{(x+2)} \quad y = 3^{(x-5)}$$

Generalize your observations by predicting what the graph of  $y = 3^{(x-c)}$  will look like in comparison to the parent function  $y = 3^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = 3^{(x-1/2)}$  will look like, and then graph it.

**Problem 4**

Graph the following equations on the same axes as the parent function  $y = 3^x$ :

$$y = 3^{2x} \quad y = 3^{1/2x}$$

Generalize your observations by predicting what the graph of  $y = 3^{cx}$  will look like in comparison to the parent function  $y = 3^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = 3^{4x}$  will look like, and then graph it.

**Step 3**

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function  $y = 3^x$ :

$$y = 3 \cdot 3^{(x+5)} \quad y = -2 \cdot 3^{(x-2)} + 1$$

After you've recorded your predictions, test them by graphing each equation.

## Handout 5D: Function Transformations Group 4

Parent function:  $y = \left(\frac{1}{3}\right)^x$

In this activity, you'll work with your peers to transform various functions you may have studied in the past. You'll explore how changes in the coefficients of a parent function affect its graph, and then examine patterns in the transformations to make conjectures about the form of a standard equation of any function. In the unit project, you'll use some of these transformations to work with equations that represent sound, and then generate tones yourself.

Your group, Function Group #4, will work on altering an equation whose graph is an exponential function. Use separate sheets of graph paper for your work.

### Step 1

Each group member should sketch a copy of the function  $y = \left(\frac{1}{3}\right)^x$ . (This is your "parent function.") Use an input/output table (also called a *T-table*) to generate values and check your graph with your peers. If a graphing utility is available, enter the function to obtain another view of this graph.

### Step 2

The following problems make four different kinds of changes to the original parent equation of the exponential function you sketched in Step 1. Have each group member complete the graphs for *one* set of equations below. (Each member needs to complete only one set of graphs, but be sure that each graph is completed by at least one member of your group.) Check one another's work—you'll be sharing it with the rest of the class.

#### Problem 1

Graph the following equations on the same axes as the parent function  $y = \left(\frac{1}{3}\right)^x$ :

$$y = \left(\frac{1}{3}\right)^x + 4 \qquad y = -1 + \left(\frac{1}{3}\right)^x$$

Generalize your observations by predicting what the graph of  $y = \left(\frac{1}{3}\right)^{x^2} + c$  will look like in comparison to the parent function  $y = \left(\frac{1}{3}\right)^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = \left(\frac{1}{3}\right)^x - \frac{1}{2}$  will look like, and then graph it.

**Problem 2**

Graph the following equations on the same axes as the parent function  $y = \left(\frac{1}{3}\right)^x$ :

$$y = -3\left(\frac{1}{3}\right)^x \qquad y = \frac{1}{2}\left(\frac{1}{3}\right)^x$$

Generalize your observations by predicting what the graph of  $y = c\left(\frac{1}{3}\right)^x$  will look like in comparison to the parent function  $y = \left(\frac{1}{3}\right)^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = -\frac{1}{4}\left(\frac{1}{3}\right)^x$  will look like, and then graph it.

**Problem 3**

Graph the following equations on the same axes as the parent function  $y = \left(\frac{1}{3}\right)^x$ :

$$y = \left(\frac{1}{3}\right)^{(x+2)} \qquad y = \left(\frac{1}{3}\right)^{(x-5)}$$

Generalize your observations by predicting what the graph of  $y = \left(\frac{1}{3}\right)^{(x-c)}$  will look like in comparison to the parent function  $y = \left(\frac{1}{3}\right)^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = \left(\frac{1}{3}\right)^{(x-1/2)}$  will look like, and then graph it.

**Problem 4**

Graph the following equations on the same axes as the parent function  $y = \left(\frac{1}{3}\right)^x$ :

$$y = \left(\frac{1}{3}\right)^{2x} \qquad y = \left(\frac{1}{3}\right)^{1/2x}$$

Generalize your observations by predicting what the graph of  $y = \left(\frac{1}{3}\right)^{cx}$  will look like in comparison to the parent function  $y = \left(\frac{1}{3}\right)^x$ , no matter what value  $c$  has. Test your idea by predicting what the graph of  $y = \left(\frac{1}{3}\right)^{4x}$  will look like, and then graph it.

**Step 3**

Work with your group to use your predictions in Step 2 to predict what the graphs of the following equations will look like, in comparison to the parent function  $y = \left(\frac{1}{3}\right)^x$ :

$$y = 3\left(\frac{1}{3}\right)^{(x+5)} \qquad y = -2\left(\frac{1}{3}\right)^{(x-2)} + 1$$

After you've recorded your predictions, test them by graphing each equation.

## Handout 6: Changing Sine

In the following activity, you'll explore transformations of the sine function and the effect of these transformations on both the graph of the function and its accompanying equation. The transformations you apply in this handout are the same transformations you worked with in Handout 5. However, here you will change the parent equation of the sine function to generate equations that can be used to represent sound waves. Look for patterns between the transformations you apply to the sine function and those you have applied to other familiar functions.

As you create the graphs, it will be useful to see at least one cycle of the sine function. To start, your  $x$ -axis should go from 0 to  $2\pi$  (around 6.3) radians. Use a scale from  $-3$  to  $3$  for the  $y$  values. Depending on the equation, you may want to adjust the  $x$ -axis scale.

If you are using a graphing utility, set its mode to radians and check the window of the graph. To view different graphs of the equations you generate, you can adjust your window.

### Part 1: The effect of $A$ in $y = A \sin(x)$

Stretch the graph of the parent function  $y = \sin(x)$  and explore the effect of changing the value of  $A$  in the equation  $y = A \sin(x)$ .

1. Sketch the graph of  $y = \sin(x)$  on graph paper.
2. Graph  $y = 2\sin(x)$  onto the same set of axes as the parent function and label it.
3. Graph  $y = \frac{1}{2}\sin(x)$  onto the same set of axes and label it. (You'll have three graphs on one set of axes.)
4. What can you conclude about the way the coefficient  $A$  in the equation above affects the graph of the basic sine function? In general, what effect does multiplying a function by a constant have on the function's graph? Use additional evidence from Handout 5 to back up your conclusion.
5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in  $A$  might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

## Part 2: The effect of $B$ in $y = \sin(Bx)$

Stretch the graph of the parent function  $y = \sin(x)$  and explore the effect of changing the value of  $B$  in the equation  $y = \sin(Bx)$ .

1. Sketch the graph of  $y = \sin(x)$  on graph paper.
2. Graph  $y = \sin(2x)$  onto the same set of axes as the parent function and label it.
3. Graph  $y = \sin\left(\frac{1}{2}x\right)$  onto the same set of axes and label it. (You'll have three graphs on one set of axes.)
4. What can you conclude about the way the value of  $B$  in the equation above affects the graph of the basic sine function? In general, what effect does multiplying the independent (input) variable by a positive constant have on the parent function's graph? Use additional evidence from Handout 5 to back up your conclusion.
5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in  $B$  might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

### Part 3: The effect of $C$ in $y = \sin(x - C)$

Stretch the graph of the parent function  $y = \sin(x)$  and explore the effect of changing the value of  $C$  in the equation  $y = \sin(x - C)$ .

1. Sketch the graph of  $y = \sin(x)$  on graph paper.
2. Graph  $y = \sin(x - \frac{\pi}{2})$  onto the same set of axes as the parent function and label it.
3. Graph  $y = \sin(x + \frac{\pi}{2})$  onto the same set of axes and label it. (You'll have three graphs on one set of axes.).
4. What can you conclude about the way the value of  $C$  in the equation above affects the graph of the basic sine function? In general, what effect does subtracting a constant from the independent variable have on the parent function's graph? Use additional evidence from Handout 5 to back up your conclusion.
5. Suppose that the graphs you created were mathematical representations of sound waves. Describe the effect that changes in  $C$  might have on the sounds you hear from each representation. Refer to your summary on Handout 1 if necessary.

### Part 4: The effect of $D$ in $y = \sin(x) + D$

Stretch the graph of the parent function  $y = \sin(x)$  and explore the effect of changing the value of  $D$  in the equation  $y = \sin(x) + D$ .

1. Sketch the graph of  $y = \sin(x)$  on graph paper.
2. Graph  $y = \sin(x) + 1$  and  $y = \sin(x) - 1$  onto the same set of axes as the parent function and label them. (You'll have three graphs on one set of axes.)
3. What can you conclude about the way the value of  $D$  in the equation above affects the graph of the basic sine function? In general, what effect does adding a constant to a function have on the function's graph? Use additional evidence from Handout 5 to back up your conclusion.

### Part 5: Understanding the general sine function

A sine function has the form  $y = A \sin(Bx - C) + D$ .

Use your understanding of function transformations to generate the graph of the following sine function:

$$y = 2 \sin\left(4x + \frac{\pi}{2}\right) - 1$$

Before creating this graph, predict how the value of each constant in the function will affect the graph of  $y = \sin x$ .

In the next activity, you'll have further opportunities to make connections between the transformed graphs and the effects of transformations on the sounds they represent.



## Handout 7: Sound Equations

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Each equation in this handout represents a sound. The variable  $x$  represents time in seconds, and  $y$  represents air pressure.

### Part 1: Predicting effects

For each of the following, use the values in the equations to predict how the sound and graph represented by the *first* equation will be different from the sound and graph represented by the *second* equation. For example, will it be louder, softer, or the same volume? How will the graph look different?

1. Compare  $y = 0.5 \sin 900x$  and  $y = 0.2 \sin 900x$ :

How will the sound generated by  $y = 0.5 \sin 900x$  be different from the sound generated by  $y = 0.2 \sin 900x$ ?

How will the graph of  $y = 0.5 \sin 900x$  be different from the graph of  $y = 0.2 \sin 900x$ ?

2. Compare  $y = 0.5 \sin 900x$  and  $y = 0.85 \sin 900x$ :

How will the sound generated by  $y = 0.5 \sin 900x$  be different from the sound generated by  $y = 0.85 \sin 900x$ ?

How will the graph of  $y = 0.5 \sin 900x$  be different from the graph of  $y = 0.85 \sin 900x$ ?

3. Compare  $y = 0.85 \sin 900x$  and  $y = 0.85 \sin 1000x$ :

How will the sound generated by  $y = 0.85 \sin 900x$  be different from the sound generated by  $y = 0.85 \sin 1000x$ ?

How will the graph of  $y = 0.85 \sin 900x$  be different from the graph of  $y = 0.85 \sin 1000x$ ?

4. Compare  $y = 0.85 \sin 900x$  and  $y = 0.85 \sin 800x$ :

How will the sound generated by  $y = 0.85 \sin 900x$  be different from the sound generated by  $y = 0.85 \sin 800x$ ?

How will the graph of  $y = 0.85 \sin 900x$  be different from the graph of  $y = 0.85 \sin 800x$ ?

5. Compare  $y = 0.85 \sin 800x$  and  $y = 0.2 \sin 900x$ :

How will the sound generated by  $y = 0.85 \sin 800x$  be different from the sound generated by  $y = 0.2 \sin 900x$ ?

How will the graph of  $y = 0.85 \sin 800x$  be different from the graph of  $y = 0.2 \sin 900x$ ?

When you have completed these problems, meet with a partner to test your predictions.

## Part 2: Testing your predictions

You'll now use sound editing software to generate audio files of these equations so you can compare them.

- Complete the following table with your partner, identifying the amplitude, period, and frequency for each equation. The first one is completed for you.

Equation	Amplitude	Period (in seconds)	Frequency (hertz, to nearest thousandth)
$y = 0.5 \sin 900x$	0.5	$\frac{\pi}{450}$ or about 0.00698	143.239
$y = 0.2 \sin 900x$			
$y = 0.85 \sin 900x$			
$y = 0.85 \sin 800x$			
$y = 0.85 \sin 1000x$			

- Check your answers to the first problem (comparing  $y = 0.5 \sin 900x$  and  $y = 0.2 \sin 900x$ ) against your partner's and discuss any disagreements you may have.
- Open the software application and do the following to generate a tone for  $y = 0.5 \sin 900x$ :
  - From the menu bar, choose **Generate > Tone . . .**
  - Be sure the Waveform is set for **Sine**, and then enter the frequency and amplitude for  $y = 0.5 \sin 900x$  from the table. Click OK.
  - Select the Zoom tool (magnifying glass). Click on the band representing the sound multiple times, until you see the thousandths place in the numbers (for example, 2.230, 2.240, 2.250, . . .). You will gradually be able to see the sound wave.
  - From the menu bar, choose **Save Project** and give it a name that will help you remember which equation was used. (For example, "Apoin5 B900" might work.) This will let you reuse the tone later rather than have to generate it again.
- Do the following to generate a tone for  $y = 0.2 \sin 900x$ :
  - From the menu bar, choose **File > New**. (This will prevent you from accidentally overwriting the other tone.)
  - Choose **Generate > Tone . . .** and enter the frequency and amplitude for the new equation. Click OK.

- Again, use the Zoom tool to zoom in until you see the thousandths place in the numbers. *Be sure both graphs have the same scales*, even if you are not looking at exactly the same time interval.
- Save the project with a name that helps you remember which equation was used. (For example, "Apoint2 B900" might work.)
- Compare the tones and graphs:
  - Play each tone—first one, then the other—by clicking the green arrow in each window. Was your prediction about how they would be different correct?
  - Now look at the graphs generated for each tone. Was your prediction about how they would be different correct?
- Use the same process to compare the other pairs (you don't need to generate a tone you've already generated, if you can remember which window belongs with which equation):
  - $y = 0.5 \sin 900x$  and  $y = 0.85 \sin 900x$
  - $y = 0.85 \sin 900x$  and  $y = 0.85 \sin 1000x$
  - $y = 0.85 \sin 900x$  and  $y = 0.85 \sin 800x$
  - $y = 0.85 \sin 800x$  and  $y = 0.2 \sin 900x$

Did you correctly predict how the sounds and graphs of these pairs would be different?

### Part 3: Conclusions

Summarize the effects of the multipliers  $A$  and  $B$  in the equation  $y = A\sin(Bx)$  on the resulting sound by completing the following:

1. When the value of  $A$  increases, the sound \_\_\_\_\_ because \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_
2. When the value of  $B$  increases, the sound \_\_\_\_\_ because \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

## Assessment Checklist: Generating Sound

Use this assessment to help you as you connect transformations of the sine equation to changes in the sound it represents. Make sure to include all the requirements. Your teacher will use this assessment to evaluate your work.

Requirements	Suggested Percentage of Total Grade	Comments	
Generating Sounds		Student Comments	Teacher Comments
Made predictions on Handout 7 before testing.	20%		
Correctly found the amplitude and period of each equation in Part 2 of Handout 7, and calculated the corresponding frequency.	35%		
Participated in class discussions.	10%		
Provided a correct conclusion for both multipliers on Handout 7, including reasoning why each has the effect it has.	35%		
<b>Total</b>	<b>100%</b>		

## Handout 8: Note Frequencies

The table below provides the frequencies for some musical notes. Notice that the frequency of the same note, but an octave higher, is doubled. You can use this relationship to find frequencies for other octaves, if needed.

You can use these frequencies in sound editing software to create your own song or to recreate one you're already familiar with. In Audacity, you can use the cursor tool to indicate where you want a tone or silence to be generated, or to select a region so you can move it.

For example, "Mary Had a Little Lamb" can be recreated using the notes C, D, E, and G:

E D C D E E E (rest or silence) D D D (rest) D G G (rest) E D C D E E E E D D E D C

You'll need to think about how much time (duration) to give each tone, and put in a small amount of silence between notes to separate them.

### Note Frequencies

Note	Frequency (Hz)
A	110.00
A <sup>#</sup> /B <sup>b</sup>	116.54
B	123.47
C (middle C)	130.81
C <sup>#</sup> /D <sup>b</sup>	138.59
D	146.83
D <sup>#</sup> /E <sup>b</sup>	155.65
E	164.81
F	174.61
F <sup>#</sup> /G <sup>b</sup>	185.00
G	196.00
G <sup>#</sup> /A <sup>b</sup>	207.65

Note	Frequency (Hz)
A	220.00
A <sup>#</sup> /B <sup>b</sup>	233.08
B	246.94
C	261.63
C <sup>#</sup> /D <sup>b</sup>	277.18
D	293.66
D <sup>#</sup> /E <sup>b</sup>	311.13
E	329.63
F	349.23
F <sup>#</sup> /G <sup>b</sup>	369.99
G	392.00
G <sup>#</sup> /A <sup>b</sup>	415.30

## Appendix A: Sound Wave Image

