Activity 1A: Ratio Calculations: Vitruvian Man

Students complete a table of ideal human proportions based on Leonardo da Vinci's iconic drawing of Vitruvian Man. Students use the ratios to calculate dimensions of body features for different-sized individuals with Vitruvian Man's proportions. Students measure dimensions of head and height for humans and animated characters and calculate their head : height ratios.

Understandings

- The relationships between body parts can be expressed mathematically using ratios.
- Human body ratios vary.
- A person's or character's appearance can be altered by varying his or her body ratios.

Materials Needed

- Optional: Printed or digital images of animated characters with varying body proportions (see *Advance Preparation*)
- **Handout 1: Unit Overview**
- **Assessment Checklist: Character Design**
- **Handout 2: Vitruvian Man Proportions**
- Optional: *Proportion Matters* slide presentation (see *Advance Preparation*)
- **Handout 3: Calculating Vitruvian Man Measurements**
- **Handout 4: Human Head : Height Ratios**
- Rulers
- Images of people and animated characters from **Appendix A: Head : Height Proportion Images** (one per student) (see *Advance Preparation*)
- **Handout 5: Journal Assignment**

1. Optional: Show students animated characters with varying proportions.
Display the images of animated characters. Ask students:

- What do you notice about the differences in the sizes of these characters' heads in relation to their bodies? What about the sizes of their facial characteristics, such as eyes and mouths?
- What effect do you think these visual properties have on your perception of the character?

2. Introduce the concept of *proportion*.
Tell students that in this unit they will look at how body and facial proportions can be quantified mathematically. They will then apply what they learn by designing an animated character with specific proportions.

Give students **Handout 1: Unit Overview** and have them read it. Distribute **Assessment Checklist: Character Design** so students can see how their work on the unit project will be assessed.

Explain that their work in this unit will help students become more accurate artists and will enable them to design characters that express clear personalities.

3. Introduce the Vitruvian Man.
Distribute **Handout 2: Vitruvian Man Proportions**. Explain that students are going to look at proportions of the human body. Tell students that Leonardo da Vinci was a 15th century engineer and architect, as well as an artist.

Teacher's Notes: Leonardo da Vinci and Vitruvian Man

Leonardo da Vinci wrote a treatise on human proportions in architecture, which included a description of ideal human body proportions as described by the Roman architect Vitruvius. See *Additional Resources for Teachers* for more information about Vitruvian Man.

Explain to students that the proportions of real human bodies vary from the ideal proportions of Vitruvian Man. Have students read the information on Handout 2.

4. Have students determine the ratios of Vitruvian Man's proportions.
On the board, draw a table similar to the table shown on Handout 2.

Note: If you are using the slide presentation *Proportion Matters*, project the first slide instead of drawing the table.

	1 finger	1 palm	1 head	1 cubit	1 height
Fingers					
Palms					
Heads					
Cubits					
Heights					

Ask students:

- What given dimensions are equal to one palm?

  Answer: *One palm is equal to four finger widths.*

Show students where to place the number 4 in the appropriate cell in the table. Work together as a class to use the information on Handout 2 to fill in the table with the other relationships described by da Vinci. Include the unit relationships in the table by writing the "1"s along the diagonal.

Note: The partially completed table should now look like this:

	1 finger	1 palm	1 head	1 cubit	1 height
Fingers	1	4			
Palms		1		6	
Heads			1		8
Cubits				1	4
Heights					1

Ask for suggestions on how to fill in the remaining cells of the table. Ask:

- How many fingers equal 1 cubit?

  Answer: *Since 4 fingers = 1 palm and 6 palms = 1 cubit, 24 fingers = 1 cubit.*

Write "24" in the appropriate cell of the table:

	1 finger	1 palm	1 head	1 cubit	1 height
Fingers	1	4		24	
Palms		1		6	
Heads			1		8
Cubits				1	4
Heights					1

Explain that the ratio of fingers to cubits is 24 to 1.

5. Have students complete the table on Handout 2.
Circulate and offer additional strategies as needed, based on your observations of student progress. If students need assistance, help them observe that the ratio between the numbers in any two columns (as well as the ratio between the numbers in any two rows) is a constant.

Point out, for example, that the ratio of fingers to palms is 4, so the numbers of fingers in the first row is always 4 times the number of palms in the second row. Similarly, point out that the ratio of cubits to height is 4, so the number of fingers, palms, and so on in the height column is always 4 times the number in the cubits column.

Once students understand the ratio between any two columns or rows, they can fill in the rest of the cells in those rows or columns of the table.

Teacher's Notes: Setting Up Proportions

You can have students set up a proportion to find the missing values in the table. For example, have students look at the four cells at the top left of the table and use the variable x to represent the unknown value:

	1 finger	1 palm
Fingers	1	4
Palms	x	1

Have students set up the proportion and solve for x:

$$\frac{1\text{ [finger]}}{x\text{ [palm]}} = \frac{4\text{ [fingers]}}{1\text{ [palm]}}$$

Remind students that the proportion can also be stated as "1 is to x as 4 is to 1."

$4x = 1$, so $x = \frac{1}{4}$.

Both numerators relate to the number of fingers, and both denominators relate to the number of palms. The ratio on the left describes the number of fingers and palms equal to 1 finger, and the ratio on the right describes the number of fingers and palms equal to 1 palm. Since the unknown value is the number of palms in 1 finger, that becomes the variable.

Students may also note that the proportion could have been set up this way:

$$\frac{1\text{ [finger]}}{4\text{ [fingers]}} = \frac{x\text{ [palms]}}{1\text{ [palm]}}$$

$1 = 4x$, so $x = \frac{1}{4}$.

With either setup, the result is the same.

Note: The completed table is given on the Teacher's Version of **Handout 2: Vitruvian Man Proportions**.

6. Discuss the table, noting the inverse relationships.
Observe with students that the table is symmetrical across the diagonal of unit relationships. Point out, for example:

- Four fingers equal one palm, and one finger is one-fourth the measure of one palm.
- Three palms equal one head, and one palm is one-third the measure of one head.
- Eight heads equal one height, and one head is one-eighth the measure of one height.

7. Have students calculate measurements using the Vitruvian proportions.
Distribute **Handout 3: Calculating Vitruvian Man Measurements**. Tell students that they are going to calculate the measurements in inches and feet of five different-sized individuals, using the Vitruvian Man proportions.

Point out that one measurement is given for each individual. Tell students to use the ratios from their completed tables on Handout 2 to help them calculate the remaining measurements on Handout 3. Have students complete Handout 3.

Teacher's Notes: Setting Up Proportions to Find Measurements

Again, students can set up a proportion to find the value of each unknown measurement. From Handout 2, students know that 4 fingers = 1 palm, and from Handout 3, they know that Individual A's finger measurement is $\frac{1}{2}$ an inch.

So, to calculate the palm measurement for Individual A, students can set up a proportion:

$$\frac{\frac{1}{2}\text{ (inch per finger)}}{1\text{ (fingers per finger)}} = \frac{x\text{ (inches per palm)}}{4\text{ (fingers per palm)}}$$

Solve for x:

$x = \frac{1}{2} \times 4$

$x = 2$

Students record a "2" in the cell for Individual A's palm on Handout 3.

Similarly, students know from Handout 2 that 3 palms = 1 head. Students can calculate the head measurement for Individual A by setting up a proportion:

$$\frac{2 \text{ (inches per palm)}}{1 \text{ (palms per palm)}} = \frac{x \text{ (inches per head)}}{3 \text{ (palms per head)}}$$

Solve for x:
$x = 2 \times 3$
$x = 6$

Students record a "6" in the cell for Individual A's head on Handout 3.

Students can continue setting up proportions to find the rest of the measurements on Handout 3.

8. Discuss students' answers to Handout 3.
Ask students to share their answers to Handout 3.

> **Note:** Students may note that not all the measurements found in Handout 3 are normal human dimensions (for example, some of the individuals are much taller than a human could be).

9. Introduce the term head : height ratio.
Have students refer to **Handout 2: Vitruvian Man Proportions**. Ask:

- How many heads tall is Vitruvian Man?

  Answer: *Eight heads tall*

Tell students that one height is the same measure as the length of eight heads. Explain that the ratio of head length to height, using heads as a unit of measurement, can be expressed as 1:8. This ratio is referred to as the head : height ratio. The *head : height ratio* of Vitruvian Man is 1:8 (in other words, he measures eight heads tall).

Ask students:

- How do you think the head : height ratios of real humans compare with Vitruvian Man?

10. Show students how to measure and calculate the head : height ratio.
Distribute **Handout 4: Human Head : Height Ratios** and give each student a ruler. Point out the images of two adults and a child. Remind students that the head : height ratio is the ratio of the head measurement to the height measurement.

Tell students that they are going to follow the steps on Handout 4 to measure each person's image and calculate the head : height ratio.

> **Note:** Tell students to measure the height of each person as if the person were standing against a wall—from the top of the head (not including the hair) to the heel.

Work together as a class to find the woman's head : height ratio.

> **Note:** The woman's head measurement is approximately 19 millimeters and her height measurement is approximately 139 millimeters. Her head : height ratio is 19:139. Because of variations in measurements, students' ratios throughout the unit may vary slightly from the answers given in this teacher guide.

Ask students:

- How can you express the ratio 19:139 in the standard ratio form of 1:*x*?

  Answer: *Set up a proportion:*

  $$\frac{19}{139} = \frac{1}{x}$$

  $$19x = 139$$

  $$x = \frac{139}{19}, \text{ or } \sim 7.3$$

  The woman's head : height ratio in standard form is 1:7.3.

> **Note:** Remind students that they can set up a proportion using three known values and a variable to represent the unknown value. They can then solve to find the value of the variable.

Have students find the head : height ratios for the images of the man and child. Discuss any differences between the adults' head : height ratios and the child's head : height ratio.

Tell students that adult humans typically have a head : height ratio from 1:6 to 1:8. In other words, adult humans range from 6 to 8 heads tall.

11. Have students calculate the head : height ratios of characters and people. Give each student one image of a person or animated character from **Appendix A: Head : Height Proportion Images** and **Handout 5: Journal Assignment**. Explain that students will measure their person's or character's height and head height and calculate the head : height ratio. Have students complete Journal 1.

Journal 1

Follow these steps to calculate the head : height ratio of the person or character your teacher gave you:

- Measure the character's head height.
- Measure the character's height.
- Calculate the head : height ratio.
- Express the head : height ratio in the standard ratio form 1:*x*.

Teacher's Notes: Measurement Precision

Students' measurements will vary slightly, depending on the precise locations of the endpoints of the line segments they drew to mark the top of the head, the bottom of the foot, and the chin. For example, the locations of the top of the head or the bottom of the foot may not be visible, so students can make an approximation.

When students express the head : height ratio in standard ratio form, suggest that they round their ratios to the nearest tenth.

Note: Journal 1 provides a good opportunity for formative assessment.