

Handout 1: Unit Overview

Optimizing Media Reach: Decision-making and Mathematics

As you prepare to stage a community event, either through your work in Foundations in Visual Arts and Media, Unit 7: Art Show! or on your own, you want as many people as possible to attend. But what kinds of promotions will bring in your potential audience? Should you create posters or place ads in the newspaper or on the radio? You probably have only a limited amount of money to spend. How can you get the best bang for your buck?

Enter mathematics—you may be surprised to learn that mathematics can help you find answers to your publicity challenge.

You'll work through a problem about maximizing media reach to promote a youth festival being organized by Wide Angle Youth Media, a non-profit organization. You'll be given an advertising budget and information about the cost and potential reach of newspaper and radio ads. You'll determine how to best use these two different media to reach the greatest number of people for the least cost.

You'll use a problem-solving strategy called linear programming and apply what you already know about linear equations and their graphs. By finding the best combination of newspaper and radio ads that reach the greatest number of people for the least cost, you'll find the answer to your publicity challenge.

Through your work on this and other problems, you will develop an understanding of the power of mathematical modeling and how you can use it to find solutions to real-world problems.

Your work in this unit will revolve around the following questions:

- *What do I need to know about a situation in order to formulate a useful problem?*
- *How can a mathematical model provide insights into a real-world problem?*
- *How can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost)?*
- *How can the linear programming process help me promote and set-up an event such as a visual arts and media event?*

Unit Portfolio

You will assemble a three-section portfolio of your work.

Section 1 will consist of your work with the class and a partner towards solving a linear programming problem about choosing promotional media. This is Problem A—Media Selection.

In Section 2, you will record your class and partner work in solving another linear programming problem. This is Problem B—Profit Maximization.

For linear programming Problems A and B, you will create:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- A resolution of the problem

In Section 3, you will record your work with a partner to formulate and explore ways to solve a linear programming problem that you create on your own. You will also use evidence from your work throughout the unit to respond to the unit's framing questions.

What You Will Do in This Unit

Find out what an objective function is. Explore objectives in real-world situations to prepare you for creating mathematical models to help solve real-world problems.

Set up a mathematical model for a linear programming problem. Organize and represent mathematically the information about Wide Angle Youth Media. Investigate how to determine the combination of newspaper ads and radio ads the group can use to achieve its objective: reaching the largest audience. Write the objective function in mathematical terms to maximize audience reach, and then graph it.

Learn about constraints in linear programming problems. Investigate a second linear programming problem related to Wide Angle Youth Media: what types of artwork should the group sell during its public event in order to maximize profit? Explore the limitations, such as cost of materials and quantity of artwork that the group can produce.

Find the best solution to each problem. Graph the constraints and use the graphs to determine how to achieve the optimum solutions of your objective functions. You'll find which combination of ads reaches the greatest audience and which combination of artworks yields the greatest profit.

Work with a partner to design your own linear programming problem. Come up with an idea for your own problem. Identify the objective function, the decision variables and constraints, and discuss ways to solve the problem.

Create a portfolio of your work. Document your work on the two linear programming problems, as well as your own problem. Reflect on your work in the unit.

Vocabulary Used in This Unit

Mathematics Terms

Constraints: In a linear programming problem, limitations of resources expressed as linear equations or linear inequalities.

Convex set: A set of points that contains an entire line segment joining any two of its points.

Decision variables: The unknown quantities that affect the objective function and the set of constraints, and frame the linear programming problem.

Dependent variable: A variable whose value *is determined by* the value of another variable.

Feasible region: The set of all ordered pairs that satisfy all of the constraints of a linear programming problem.

Function: A rule that maps each element in one set to exactly one element in a second set.

Independent variable: A variable whose value *determines* the value of another variable

Infeasibility: The situation in which no solution to a linear programming problem satisfies all of the constraints.

Linear combination: A sum of multiples of linear equations.

Linear equation: An algebraic equation in which each term is either a constant or a constant multiplied by the first power of a single variable. The standard form of a linear equation can be written: $ax + by = c$, where a , b , and c are constants, and x and y are variables.

Linear programming: A problem-solving approach that optimizes an objective function given limitations to resources (constraints). Both the objective function and the constraints can be represented with linear equations and/or linear inequalities.

Objective function: A mathematical expression that represents the goal of maximizing or minimizing a particular quantity when there are limited resources. The objective function is a measure of effectiveness that makes possible the comparison of feasible solutions.

Optimum solution: The feasible solution that provides the best possible value of the objective function.

Media-related Terms

Circulation: The total number of copies of a publication sold through various forms of distribution.

Exposure quality rating: A measure of the relative value of one advertisement in a specific medium. This measure takes into account factors such as audience demographics (e.g., age, income, education, race), image presented, and quality of the advertisement.

Media: The channels by which ads are carried to a target market.

Media objective: A statement in the media plan that explains the goals of the plan, often stating how many people within the target audience will be exposed to advertising messages in a given time period and how often.

Media plan/media schedule: A document that establishes how media will be used to disseminate the advertiser's message, including goals (objectives) and strategies.

Media reach: The percent of a target audience that can potentially be exposed to a particular media plan in a given time frame.

Media vehicle: Type of medium used to disseminate an advertiser's message. Some examples are radio, television, newspaper, posters, online sources, and bulletin boards.

Handout 2: Problem A—Media Selection

Introduction to Wide Angle Youth Media

Wide Angle Youth Media is a non-profit organization in Baltimore, Maryland. It provides media tools and education in video and audio production so that students can communicate messages to their community. Students produce and showcase their own work.

Wide Angle Youth Media reaches audiences in communities around the United States through online channels such as *Facebook* and *YouTube* and through partnerships with other cultural organizations. However, the audience in the local community is small, mostly composed of people directly connected to the organization, such as family members, board members, and community advocates.

Wide Angle Youth Media has taken on the challenge of widening its audience to reach more members of the Baltimore community. The group created a new mission and developed a festival to address the mission.

The “*Who Are You?*” Youth Media Festival is an event framed around the theme of *identity*. The festival is a collaboration among Wide Angle Youth Media and other nonprofit groups that serve youth in Baltimore. It includes not only video and audio, but also photography, fine art, poetry, live performances, and a gallery exhibit.

Promoting the Festival

Wide Angle Youth Media wants to reach beyond its usual audience and involve community members in the festival. So Wide Angle Youth Media has set up a promotion committee and a budget. The committee’s task is to reach as many people as possible during the three weeks prior to the festival. Wide Angle’s executive director wants to use only two types of media to promote the event.

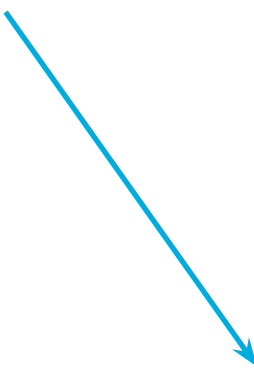
As part of the promotion committee, you will help Wide Angle Media create a plan that *optimizes reach*—that is, reaches as many potential audience members as possible while staying within the given budget.

Handout 3: Objective Match-Up

Part 1

Match each business or organization at the left with an objective at the right.

Business/Organization	Objective
United States Treasury	To maximize sales of a new model
An airline	To maximize knowledge and understanding
A car manufacturer	To maximize the number of surgical operations performed each day
A coffee shop	To maximize tax revenue
A consumer	To maximize the number of cases handled yearly
A mathematics department in a school	To maximize satisfaction through the consumption of goods and services
A hospital	To minimize fuel costs
A law firm	To maximize use of floor space



Materials in this handout and activity have been adapted from the *METAL (Mathematics for Economics: enhancing Teaching and Learning) Guide 4: Linear Programming*, by S. D. Hawkins. Content is licensed under a Creative Commons Attribution-Non-Commercial 2.0 UK: England & Wales License.

Part 2

For each organization in the left column of the table below, write its objective in the middle column. You can use the objective you identified in Part 1.

With a partner, brainstorm some limitations that might affect the objective of each organization. Write the limitations, or constraints, in the right column.

You might ask yourself, "What kinds of constraints or limitations could influence the objective of the business?" An example of constraints is provided below.

Organization	Objective	Constraints
An airline		
A car manufacturer		
A hospital		
<i>Example:</i> A consumer	To maximize satisfaction through the consumption of goods and services	A fixed income Prices of goods Quantity of goods available
A coffee shop		

Handout 4:

Problem A—Media Selection: More Information

The Wide Angle Youth Media promotion committee for the “Who Are You?” Youth Media Festival wants to reach as many people as possible in the Baltimore area during the three-week period prior to the festival.

The Executive Director has allocated a budget for advertising. She has asked the promotion committee to limit advertising to two media vehicles, a local newspaper and a radio station. The promotion committee gathered the additional information below.

1. The advertising budget of \$4,000 is to be used to promote the festival over the three-week period prior to the event.
2. For both newspaper ads and radio spot ads, the committee needs to know:
 - Number of potential customers reached through the medium
 - Cost per advertisement
 - Maximum number of times each medium is available during a time period
 - Audience demographics
3. Media kits on the Web sites for the newspaper and radio station give the cost of ads, as well as the number of potential customers reached through one ad.

Local newspaper: A daily one-quarter page ad costs \$70. The daily circulation is estimated at 200,000 people; the committee believes that only 1% of this number of people will respond to the ad and attend the festival for each day the ad runs.

Local radio station: A set of three 30-second radio ads broadcast in one day costs \$200. The total population of Baltimore is approximately 600,000. It is estimated that three daily broadcast spots reach about 3% of that population. Of this portion of the population, the committee expects that 22% will hear the radio ads and decide to attend the festival for each day the ads run.

4. Festival sponsors from the radio station and newspaper have placed one additional limitation on the use of the two media vehicles. They have asked that the number of days the ads run be distributed somewhat evenly between the two media. The limitation is described below:

The ratio of the number of days the radio ads run to the number of days the newspaper ads run must be between $\frac{1}{2}$ and 2. The sponsors recognize that if the ratio were 1, the limitation would be very restrictive; it would mean that the ads must run for the same number of days on each media vehicle. Instead, the sponsors are willing to be flexible by having this ratio range between $\frac{1}{2}$ and 2.

Handout 5: Linear Functions and Their Representations

You can represent the additional information in Problem A—Media Selection with expressions and linear equations. Use your knowledge of functions and linear equations to help you solve the problems below.

1. The promotion committee wants to understand further what the information in the newspaper media kit means. Consider the information given:

A daily one-quarter page ad in the local newspaper costs \$70. The daily circulation of the paper is estimated at 200,000 people; the committee believes that only 1% of this number of people will respond to the ad and attend the festival for each day the ad runs.

Find the estimated reach (the number of people who may likely attend the festival if they see the newspaper ad) of a one-quarter page ad in the local newspaper.

2. Similarly, the promotion committee wants to understand further what the information in the radio station media kit means. Consider the information given:

A set of three 30-second radio ads broadcast in one day costs \$200. The total population of Baltimore is approximately 600,000. It is estimated that three daily broadcast spots reach about 3% of that population. Of this portion of the population, the committee expects that 22% will hear the radio ads and decide to attend the festival for each day the ads run.

Find the estimated reach (the number of people who may likely attend the festival if they hear the radio spot ads) of a set of 3 radio spot ads broadcast in one day.

3. Use your findings from Problems 1 and 2 to determine the combination of newspaper ads and sets of radio spot ads that will reach exactly 20,000 people who will likely attend the festival.
- Assign variables. What are the unknown quantities? Use letters to represent these variables.
 - Look for relevant information. What do you know about the estimated reach of each medium?
 - Write an equation that represents the combinations of radio spot ads and newspaper ads that together reach exactly 20,000 people likely to attend the festival.
 - Graph the equation using the intercepts.
 - Interpret the meaning of the graph.

Handout 6:

Assembling Your Portfolio

You will learn the principles behind linear programming as the class works on Problem A—Media Selection. You will also apply the linear programming approach to another problem, Problem B—Profit Maximization. In addition, you will work with a partner to formulate your own linear programming problem and discuss ways to solve it.

You will assemble all of your work in a three-section portfolio.

Portfolio Requirements

Section 1: Problem A—Media Selection

Gather work done on Problem A—Media Selection. Be sure to include:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- Resolution of the problem

Section 2: Problem B—Profit Maximization

Gather work done on Problem B—Profit Maximization. Be sure to include:

- A problem statement
- A mathematical representation of the objective function for the problem
- A set of linear inequalities representing the constraints of the problem
- A graphical solution displaying the feasible region
- Resolution of the problem

Section 3: Problem Formulation and Written Reflection

Document your work with a partner on an idea for your own linear programming problem. Be sure to include:

- A problem statement
- A statement of the objective function
- Your choice for possible decision variables
- Constraints in terms of the decision variables chosen

Write a response to the framing questions below, using examples from the work you did throughout the unit:

- What do I need to know about a situation in order to formulate a useful problem?
- How can a mathematical model provide insights into a real-world problem?
- In particular, how can I use linear equations and inequalities to determine the best possible value for a quantity (such as profit or cost) that is dependent on variables I can change?

Assessment Checklist: Unit Portfolio

Use this checklist to help you assemble your portfolio. Make sure to include all the requirements. Your teacher will use this assessment to evaluate your work.

Requirements	Percentage of Total Grade	Comments	
Section 1.			
Problem A—Media Selection	40%	Student Comments	Teacher Comments
Mathematical model includes an explanation of the decision variables.			
The objective function is a linear expression stated in terms of the decision variables.			
The set of constraints is expressed as linear inequalities in terms of the decision variables.			
A graphical solution displays the feasible region.			
Corner points are identified and the <i>best</i> solution determined.			
Section 2.			
Problem B—Profit Maximization	40%	Student Comments	Teacher Comments
Mathematical model includes an explanation of the decision variables.			
The objective function is a linear expression stated in terms of the decision variables.			
The set of constraints is expressed as linear inequalities in terms of the decision variables.			

A graphical solution displays the feasible region.			
Corner points are identified and the <i>best</i> solution determined.			

Section 3.

Problem Formulation and Written Reflection	20%	Student Comments	Teacher Comments
Partner work: The problem statement generated can be resolved by using a linear programming approach.			
Partner work: The objective is clearly explained and justified.			
Partner work: The objective and the constraints depend on two decision variables.			
Individual reflection: Evidence from work is used to support responses to the framing questions.			
Individual reflection: Response is well organized and addresses the framing questions clearly and thoroughly.			
Total	100%		

Handout 7: Problem B—Profit Maximization

You and your partner are going to create a mathematical model for a linear programming problem about maximizing profit. You are going to:

- determine the decision variables
- write the objective function in terms of the decision variables
- identify the constraints on the decision variables
- organize the constraints in a table
- graph the inequalities that represent each constraint

Read the linear programming problem below and then follow the steps to work towards a solution for the problem.

The Problem

You plan to sell two kinds of artwork at the Youth Media Festival and donate the proceeds to Wide Angle Youth Media for future work in youth media education.

You want to figure out how many drawings and how many mixed-media collages to make prior to the festival. You can spend up to \$240 on supplies. The cost of supplies for one drawing is \$8. The cost of supplies for one mixed-media collage is \$16. You have enough time to complete at most 20 pieces of artwork.

If each drawing makes a profit of \$40 and each mixed-media collage makes a profit of \$65, how many of each type of artwork should you create to maximize the amount of money received from sale of the artwork?

Step 1: Set up the problem.

1. State the objective of the problem in your own words.

2. The unknown quantities in this problem are the number of drawings and the number of mixed media collages. Assign the variable x to represent the number of drawings and y to represent the number of mixed media collages.

- How are your decision variables related to each other?
- Write your objective in terms of your decision variables. Write a mathematical expression that represents the goal you stated in #1 above, using the variables you chose in #2 above. Note that the objective will be an expression rather than an equation.
- List the constraints that affect your production of artwork.

Step 2: Organize the constraints.

Use the table below to record information about the constraints you identified.

Table of Constraints			
	Drawings x	Mixed-media collages y	Quantities of Input Available
Time (number of art pieces)			
Cost of supplies			
Number of drawings			
Number of mixed-media collages			

Step 3: Work with each constraint.

Your teacher will provide you with graph paper or gridded transparencies on which you will create graphs as you work with each constraint.

Constraint 1: Time

Plot all of the possible combinations of drawings and mixed-media collages that add up to a total of 20. Label the x -axis using your independent variable and the y -axis using your dependent variable.

- What kind of function do you obtain? Explain.
- Is it possible to make fewer than 20 pieces? Shade the part of the graph that represents 20 art pieces or fewer.
- What inequality could you write to represent this relationship?
- How could you obtain this inequality using the table of constraints from Step 2 rather than the graph you created?

Constraint 2: Cost of supplies

Follow the steps below to create a graph that represents the financial constraint on supplies.

- Write an expression that represents the money you will spend on supplies if you create x drawings.
- Write an expression that represents the money you will spend on supplies if you create y mixed-media collages.
- Write an *equation* that shows that the total amount of money you will spend on supplies when creating x drawings and y mixed-media collages is *exactly* \$240.
- Explain why the equation you wrote is a linear equation. Graph the equation on a set of axes. (Remember that you can use the x - and y -intercepts when graphing). Keep the labeling consistent with your previous graph.
- Write an *inequality* that expresses the financial constraint on supplies.
- Graph the inequality. The inequality will split the plane into two regions.

Choose a point in one of the two regions of the plane and test its coordinates in the inequality. If the point satisfies the inequality, shade the part of the graph that includes that point. If the point doesn't satisfy the inequality, choose a point in the other region of the plane to test.

The part of the graph that satisfies the inequality is the solution set for this constraint. Shade this part of the graph.

Constraint 3: Number of drawings

Look at your table of constraints. Write an inequality to express the constraint on the number of drawings.

Note that this inequality, as well as the inequality that represents the constraint on the number of collages, can be written as a *one-variable* inequality. Recall that you can also represent one-variable inequalities on the two-dimensional coordinate plane.

Graph this inequality.

Constraint 4: Number of mixed-media collages

Write an inequality to express the constraint on the number of collages. Remember that you can write this constraint as a *one-variable* inequality.

Graph this inequality.

You have created a mathematical model for a linear programming problem. Later in the unit, you will use this model to find the *optimal solution* for Problem B—Profit Maximization.

Handout 8: Problem B—Profit Maximization: The Feasible Region

You developed a mathematical model for Problem B through your work on **Handout 7: Problem B—Profit Maximization**.

Fill in the model using your work from Handout 7. Then answer the follow-up questions on this handout to work toward finding the solutions to Problem B—Profit Maximization.

Problem B—Profit Maximization
Mathematical Model

Decision variables:

Objective, with Objective Functions:

Constraints:

Additional restrictions:

Follow-up Questions

Use the mathematical model and your completed graph showing all of the constraints to respond to the questions below.

1. Label a few points on the boundary of the feasible region and interpret what these particular points indicate.
2. Label a few points on the outside of the feasible region and interpret what these particular points indicate.
3. Could you create 10 drawings and 4 mixed-media collages while satisfying all of the constraints of the problem? Explain your response.
4. Could you create 4 drawings and 14 collages and satisfy all of the constraints of the problem? Why or why not?

5. Could you create 15 drawings and 2 collages and satisfy all of the constraints? If so, what profit would you make from selling this combination of artwork pieces, assuming that all of them can be sold?

Hint: Use the given values to evaluate the objective function.

6. Which combination will give you a higher profit: 10 drawings and 4 collages or 5 drawings and 5 collages?

7. Are there other possible combinations of drawings and mixed-media collages that satisfy all of the constraints in the problem? Explain.

Handout 9: Problem A—Media Selection: Mathematical Model

The mathematical model for Problem A is given below. Find all of the possible solutions, or the feasible region, for the problem.

First, use the model to create a graph. Then answer the follow-up questions on this handout. Your teacher will check your work.

Problem A—Media Selection Mathematical Model

Decision variables:

r = number of sets of 3 radio spot ads to run during the three-week promotion period

n = the number of newspaper ads to run during the promotion period

Objective:

To determine the number of radio spot ads and the number of newspaper ads that maximize media reach subject to specific constraints.

Reach is represented by the expression $3,960r + 2,000n$

Constraints:

Non-negativity: $r \geq 0, n \geq 0$

Budget: $200r + 70n \leq 4,000$

Time limit for promotion: $r \leq 21$ and $n \leq 21$

Additional restrictions on budget:

$$\frac{1}{2} \leq \frac{r}{n} \leq 2$$

This inequality can be expressed in two parts:

$$n \geq \frac{1}{2}r \text{ and } n \leq 2r$$

Follow-up Questions

1. Label a few points on the boundary of the feasible region you found and interpret what these particular points indicate.
2. Label a few points on the outside of the feasible region and interpret what these particular points indicate.
3. Could you broadcast 8 sets of 3 radio spot ads and purchase 10 newspaper ads while satisfying all of the constraints of the problem? Explain.

4. Could you place 10 sets of radio spot ads and 10 newspaper ads and still meet all of the constraints? If so, how many people will you reach?
Hint: Use the given values to evaluate the objective function.
5. Which combination will reach more people: 10 sets of radio spot ads and 10 newspaper ads or 11 sets of radio spot ads and 9 newspaper ads? Why does this make sense given the context of this problem?
6. Are there other possible combinations of radio spots and newspaper ads that satisfy all of the constraints?

Handout 10: Problem B—Profit Maximization: Optimal Solution

You are going to work toward finding the optimal solution for Problem B—Profit Maximization. Use your mathematical model for Problem B, along with the graph you created that shows the feasible region for Problem B.

Your teacher will help you and your partner complete Problem 1. Work together with your partner to complete Problem 2.

1. Recall that the objective function for Problem B is an expression that represents maximizing profit: $40x + 65y$.

Set the objective function for Problem B equal to 0. Solve for y . Graph this linear equation.

- What is the slope of the line?
- What is the y -intercept?
- What does any point on this line represent?

Now set the objective function for Problem B equal to \$100. Solve for y once again. Graph this linear equation.

- What is the slope of this second line?
- What is the y -intercept?
- What does any point on this line represent?

2. Continue to increase the value of the profit by setting the objective function for Problem B at amounts greater than \$100. Do this for at least three new profit values, in increasing order.

Graph the linear equations. Overlay your graph on the graph of the feasible region for Problem B.

Use your graphs to answer the questions below:

- The lines you graphed are called *constant profit lines*. What is the relationship among these lines? How are they similar? How are they different?

- An infinite number of points lie on each constant profit line. How can you determine which points satisfy the constraints of Problem B? What do these points represent?

- Remember that you are trying to find the optimal solution to Problem B—Profit Maximization. This solution makes the value of the objective function as great as possible (to maximize profit) while still satisfying the constraints of the problem.

Use your graphs to help you visualize what happens as profit increases. Think about the location of the constant profit lines and the location of the points on those lines that meet the problem's constraints. Then write a conjecture about the location of the optimal solution to Problem B.

The Fundamental Theorem of Linear Programming

Answer the following question: What is the Fundamental Theorem of Linear Programming?

Handout: Solving Systems of Linear Equations

You are going to learn about the elimination method for solving systems of equations. You can apply this method to find the coordinates of the corner points in the feasible region for Problem A—Media Selection. Then you can go on to solve Problem A.

Use a separate sheet of paper for your calculations and one sheet of graph paper with coordinate axes.

Solve the following system of two equations with two unknown variables:

$$5x - 2y = 10 \quad (1)$$

$$x - y = -1 \quad (2)$$

1. What does it mean to solve a system of two equations? Recall that solving one equation with one unknown variable means finding the value(s) that makes the equation true.

2. Graph the two linear equations on the same coordinate plane. It may be helpful to use x - and y -intercepts when graphing each line.

3. Line up the two equations and add them, term by term. Graph the resulting equation on the same coordinate plane. Be sure to label each line on your graph.

4. Multiply equation (2) by 3 and add your result to equation (1). Graph this equation on the same coordinate plane.

5. Multiply (2) by -2 and add the result to equation (1). Graph this equation on the same coordinate plane.

6. As a last step in solving this system of equations, multiply equation (2) by -5 and add the result to equation (1). Why is this the last step?

Graph the new equation and record your observations. Review your work in Steps 1 through 6 and prepare to make generalizations about the elimination process in Step 7 below.

7. Each line that you graphed is a *linear combination* of the two lines with which you started. A *linear combination* is a sum of multiples of linear equations. You obtained each new line by multiplying one of the original lines and adding it to the other.
- (a) What is common among all the linear combinations of the two lines?
- (b) Do you think this is true for all cases?
- (c) What is the solution of the problem on this handout? Which of the equations you generated is satisfied by the solution?
- (d) When solving a system of linear equations, some combinations are more useful than others. Which linear combinations were useful in obtaining the point of intersection?
- (e) Write a procedure that explains the process and rationale of the elimination method. If necessary, begin with an example and explain the method as you proceed through its various steps.